INSTRUCTIONS FOR PART I: Write your answers for these questions on a scantron (form 882-ES or 882-E) and mark only one answer per question.

Each of the questions in this part counts 5 points each, for a total possible score of 50 points. You may use an approved calculator. You may write on this exam or request scratch paper if needed.

1. Find a formula for the $n$th partial sum of the series and use it to compute the series’ sum if it converges.

$$
\sum_{n=1}^{\infty} \left( \frac{6}{n(n+1)} \right) = \frac{6}{1\cdot2} + \frac{6}{2\cdot3} + \frac{6}{3\cdot4} + \cdots + \frac{6}{n(n+1)} + \cdots
$$

(A) $\frac{7}{9}$ (B) $\frac{6(n+2)}{(n+1)}$; 6 (C) $\frac{6(n+1)}{n+2}$; 6 (D) $\frac{6n}{n+1}$; 6 (E) none of these

2. Find the sum as a function of $x$: $\sum_{k=0}^{\infty} \left( \frac{x-9}{10} \right)^k$.

(A) $\frac{10}{x+9}$ (B) $\frac{-10}{x-19}$ (C) $\frac{-10}{x+19}$ (D) $\frac{10}{x-19}$ (E) none of these

3. Find the Taylor polynomial of order 3 generated by $f(x) = \ln(x+1)$ at $a = 4$.

(A) $\ln 5 + \frac{x-4}{3} - \frac{(x-4)^2}{18} + \frac{(x-4)^3}{81}$ (B) $\ln 3 - \frac{x-4}{3} - \frac{(x-4)^2}{18} + \frac{(x-4)^3}{81}$

(C) $\ln 5 + \frac{x-4}{5} - \frac{(x-4)^2}{50} + \frac{(x-4)^3}{375}$ (D) $\ln 5 - \frac{x-4}{5} + \frac{(x-4)^2}{50} - \frac{(x-4)^3}{375}$ (E) none of these

4. Which of the following represent volumes of solids of revolution revolved about the $x$-axis?

I. $\pi \int_{a}^{b} \left[ f(x) \right]^2 \, dx$  II. $2\pi \int_{a}^{b} \left[ f(x) - g(x) \right] \, dx$  III. $\pi \int_{a}^{b} \left[ \left( f(x) \right)^2 - \left( g(x) \right)^2 \right] \, dx$

(A) II only  (B) I and II  (C) I and III  (D) II and III  (E) I, II, and III

5. For what value of $x$ does the series $\sum_{k=1}^{\infty} \frac{(x-3)^k}{k}$ converge conditionally?

(A) $x = 2$  (B) $x = 4$  (C) $x = 1$  (D) $x = -1$  (E) $x = 3$
6. Find a unit vector parallel to $\overrightarrow{PQ}$ where $P = (0, 3, -2)$ and $Q = (4, 2, 1)$.

(A) $\langle 4, -1, 3 \rangle$  (B) $\frac{1}{\sqrt{18}} \langle 4, -1, -1 \rangle$  (C) $\frac{1}{\sqrt{13}} \langle 0, 3, -2 \rangle$  (D) $\frac{1}{\sqrt{21}} \langle 4, 2, 1 \rangle$

(E) $\frac{1}{\sqrt{26}} \langle 4, -1, 3 \rangle$

7. The vertices $A, B$ and $C$ of a triangle in $\mathbb{R}^3$ are given by: $A(1, 1, 1), B(3, 3, 2), C(3, -3, 5)$. The triangle is a(n)

(A) equilateral triangle  (B) right triangle  (C) isosceles triangle  (D) isosceles and right triangle  (E) none of these

8. The lines given by $\frac{x-4}{2} = \frac{y-6}{-3} = \frac{z+2}{5}$ and $\frac{x}{4} = \frac{y+2}{-6} = \frac{z-3}{10}$

(A) intersect at the point $(4, 6, -2)$  (B) intersect at the point $(0, -2, 3)$

(C) are coincident  (D) are parallel  (E) are skew

9. The approximate angle, in radians, between the vectors $\mathbf{u} = \langle 4, 1, 3 \rangle$ and $\mathbf{v} = \langle -3, 6, 1 \rangle$ is

(A) 1.658  (B) -0.0867  (C) -4.977  (D) 86.94  (E) -3

10. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 0$, $x = 1$, and $x = 2$ about $x = 1$.

(A) $2\pi$  (B) $\frac{17\pi}{6}$  (C) $3\pi$  (D) $\frac{27\pi}{6}$  (E) $5\pi$

INSTRUCTIONS FOR PART II: For these questions, you must write down all steps in your solutions. Write legibly and carefully label any graphs or pictures. Draw a box around your solution. Partial credit will be given for those parts of your solution that are correct. Each of the questions in this part counts 10 points, for a total possible score of 50 points.

11. Compute the interval of convergence of the power series $\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{\sqrt{k^2 + 1}}$.

12. Determine the convergence or divergence of the series $\sum_{k=0}^{\infty} k e^{-k}$.

13. Find an equation of the plane that passes through the points $(0, 2, -1)$, $(1, -3, 5)$, and $(3, 0, -2)$.

14. Evaluate the integral $\int x (\ln x)^2 \, dx$.

15. Find the value of $\int_{0}^{e} \frac{1}{x-2} \, dx$. 

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