Quiz #1 (Spring 2007)  
February 16, 11:00-12:00pm

Throughout the quiz, assume air is an ideal gas and has constant specific heats.

Thermal properties of Air:  
R = 0.287 kJ/kgK  
C_p = 1.005 kJ/kgK  
C_v = 0.718 kJ/kgK

1. (30 Points) An Otto cycle has a compression ratio of 8 and maximum temperature of 627°C. At the beginning of the compression stroke, the pressure and temperature of the working fluid are 100 kPa and 27°C. (a) Draw the cycle in P-v diagram with indicating the given conditions. (b) Determine how much net work can be produced per cycle with unit mass of air.

2. (30 Points) At the start of the compression stroke of a Diesel cycle, the air is at 100 kPa, 27°C. This cycle has a compression ratio of 16, and a heat input of 400 kJ/kg. (a) Draw the cycle in P-v diagram with indicating the given conditions. (b) Determine the cut-off ratio for this cycle.

3. (40 Points) Consider a gas turbine cycle based on an ideal Brayton cycle. Air enters the compressor at 100kPa and 25°C. The pressure ratio of the cycle is 5, and the turbine inlet temperature is 850°C. (a) Draw the cycle in T-s diagram with indicating the given conditions. Determine (b) the net work per unit mass flow rate, and (c) the thermal efficiency of the cycle.
$T_1 = 627 = 900 \, K$

$T_2 = 37 + 273 = 300 \, K$

$p_i = 100 \, \text{psi}$

$$w_{net} = q_{in} - q_{out}$$

$$q_{in} = c_v (T_3 - T_2) = 0.718 \times (900 - 689.21) = 151.34 + 5$$

$$q_{out} = c_v (T_4 - T_1) = 0.718 \times (391.7 - 300) = 65.84 + 5$$

$$\frac{T_4}{T_1} = \left(\frac{V_2}{V_1}\right)^{k-1}$$

$$T_2 = \frac{T_1}{\left(\frac{V_2}{V_1}\right)^{k-1}} = \frac{300}{0.81} = 391.7 \, K$$
2. \( \frac{P_1}{T_1} = \frac{P_2}{T_2} \)  

\( T_1 = 97^\circ = 360 \, K \)  

\( r_c = \frac{r_1}{r_2} \)  

\( v_1 = V_4 \)  

\[ q_i = 400 \frac{K}{V_2} = C_p \left( T_3 - T_2 \right) \]

\[ r_c = \frac{V_2}{V_1} \]

\[ T_3 = \frac{T_1 \left( \frac{V_2}{V_1} \right)^{k-1}}{\left( \frac{1}{16} \right)^{k-1}} = 907.43 \, K \]  

\[ q_i = 400 \left( T_3 - 907.43 \right) \]

\[ T_3 = 1307.44 \, K \]  

\[ \frac{V_2}{V_3} = \frac{T_2}{T_3} \]  

\[ \frac{T_2}{T_3} = \frac{1307.44}{907.43} = 1.4 \]  

\[ +3 \]

\[ +7 \]

\[ +5 \]

\[ +5 \]
\[ T_1 = 25^\circ C = 298\, \text{K} \]
\[ P_1 = 100\, \text{kPa} \]
\[ T_1 = 5 = \frac{P_2}{P_1} = \frac{P_3}{P_4} \quad \Rightarrow \quad P_2 = P_3 \quad \Rightarrow \quad P_1 = P_4 \]
\[ T_3 = 850^\circ C = 1123\, \text{K} \]

(b) \[ \frac{m}{m_{\text{turb}}} = \frac{w_{\text{turb}}}{w_{\text{comp}}} \]
\[ w_{\text{turb}} = c_p(T_5 - T_7) = 4.005(1123 - 709.09) = 416.02 \]
\[ w_{\text{comp}} = c_p(T_9 - T_1) = 4.005(771.978 - 298) = 174.85 \, \text{kJ} \]

\[ T_2 = \left( \frac{P_3}{P_1} \right)^{\frac{K-1}{K}} = \left( \frac{P_3}{P_4} \right)^{\frac{K-1}{K}} = \frac{T_3}{T_4} \]
\[ T_2 = T_1 \left( \frac{P_3}{P_1} \right)^{\frac{K-1}{K}} = 298 \left( \frac{5}{1} \right)^{\frac{1.4-1}{1.4}} = 471.978 \, \text{K} \]
\[ T_4 = \frac{1}{T_3} \left( \frac{P_3}{P_4} \right)^{\frac{K-1}{K}} = 1123 \left( \frac{1}{5} \right)^{\frac{1.4-1}{1.4}} = 709.04 \, \text{K} \]

\[ \frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_5}{T_1} = 709.04 \, \text{K} \]

\[ m = \frac{P V}{R T} \]

(c) \[ \eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{IN}}} = 1 - \frac{\text{out}}{\text{in}} \]
\[ \eta_{\text{th}} = 1 - \frac{c_p(T_3 - T_7)}{c_p(T_5 - T_7)} = 1 - \frac{(1123 - 709.09)}{(771.978 - 298)} \]
\[ \eta_{\text{th}} = 0.8686 \]

40/40