Solutions to HW # 5

7-82E The sink temperature of a Carnot heat engine, the rate of heat rejection, and the thermal efficiency are given. The power output of the engine and the source temperature are to be determined.

Assumptions The Carnot heat engine operates steadily.

Analysis (a) The rate of heat input to this heat engine is determined from the definition of thermal efficiency,

\[
\eta_{th} = 1 - \frac{Q_L}{Q_H} \quad \Rightarrow \quad 0.75 = 1 - \frac{800 \text{ Btu/min}}{Q_H} \quad \Rightarrow \quad Q_H = 3200 \text{ Btu/min}
\]

Then the power output of this heat engine can be determined from

\[
\dot{W}_{net, out} = \eta_{th} \dot{Q}_H = (0.75)(3200 \text{ Btu/min}) = 2400 \text{ Btu/min} = \mathbf{56.6 \text{ hp}}
\]

(b) For reversible cyclic devices we have

\[
\left(\frac{\dot{Q}_H}{\dot{Q}_L}\right)_{\text{rev}} = \left(\frac{T_H}{T_L}\right)
\]

Thus the temperature of the source \(T_H\) must be

\[
T_H = \left(\frac{\dot{Q}_H}{\dot{Q}_L}\right)_{\text{rev}} T_L = \left(\frac{3200 \text{ Btu/min}}{800 \text{ Btu/min}}\right)(520 \text{ R}) = \mathbf{2080 \text{ R}}
\]

7-88 A geothermal power plant uses geothermal liquid water at 160°C at a specified rate as the heat source. The actual and maximum possible thermal efficiencies and the rate of heat rejected from this power plant are to be determined.

Assumptions 1 The power plant operates steadily. 2 The kinetic and potential energy changes are zero. 3 Steam properties are used for geothermal water.

Properties Using saturated liquid properties, the source and the sink state enthalpies of geothermal water are (Table A-4)

\[
\begin{align*}
T_{\text{source}} &= 160^\circ \text{C} \\
x_{\text{source}} &= 0 \\
h_{\text{source}} &= 675.47 \text{ kJ/kg}
\end{align*}
\]

\[
\begin{align*}
T_{\text{sink}} &= 25^\circ \text{C} \\
x_{\text{sink}} &= 0 \\
h_{\text{sink}} &= 104.83 \text{ kJ/kg}
\end{align*}
\]

Analysis (a) The rate of heat input to the plant may be taken as the enthalpy difference between the source and the sink for the power plant

\[\dot{Q}_m = \dot{m}_{geo}(h_{\text{source}} - h_{\text{sink}}) = (440 \text{ kg/s})(675.47 - 104.83) \text{ kJ/kg} = 251,083 \text{ kW}\]

The actual thermal efficiency is

\[
\eta_{th} = \frac{\dot{W}_{net, out}}{\dot{Q}_m} = \frac{22 \text{ MW}}{251,083 \text{ MW}} = \mathbf{0.0876 = 8.8\%}
\]

(b) The maximum thermal efficiency is the thermal efficiency of a reversible heat engine operating between the source and sink temperatures
\[ \eta_{\text{th,max}} = 1 - \frac{T_L}{T_H} = 1 - \frac{(25 + 273) \text{K}}{(100 + 273) \text{K}} = 0.312 = 31.2\% \]

(c) Finally, the rate of heat rejection is
\[ Q_{\text{out}} = Q_{\text{in}} - W_{\text{net, out}} = 251.1 - 22 = 229.1 \text{ MW} \]

7-99 The refrigerated space temperature, the COP, and the power input of a Carnot refrigerator are given. The rate of heat removal from the refrigerated space and its temperature are to be determined.

**Assumptions** The refrigerator operates steadily.

**Analysis** (a) The rate of heat removal from the refrigerated space is determined from the definition of the COP of a refrigerator,
\[ Q_L = \text{COP}_{R_{\text{rev}}} W_{\text{net, in}} = (4.5)(0.5 \text{ kW}) = 2.25 \text{ kW} = 135 \text{ kJ/min} \]

(b) The temperature of the refrigerated space \( T_L \) is determined from the coefficient of performance relation for a Carnot refrigerator,
\[ \text{COP}_{R_{\text{rev}}} = \frac{1}{(T_H / T_L) - 1} \rightarrow 4.5 = \frac{1}{(25 + 273 \text{ K})/T_L - 1} \]
It yields
\[ T_L = 243.8 \text{ K} = -29.2^\circ \text{C} \]

7-101E An air-conditioning system maintains a house at a specified temperature. The rate of heat gain of the house and the rate of internal heat generation are given. The maximum power input required is to be determined.

**Assumptions** The air-conditioner operates steadily.

**Analysis** The power input to an air-conditioning system will be a minimum when the air-conditioner operates in a reversible manner. The coefficient of performance of a reversible air-conditioner (or refrigerator) depends on the temperature limits in the cycle only, and is determined from
\[ \text{COP}_{R_{\text{rev}}} = \frac{1}{(T_H / T_L) - 1} = \frac{1}{(95 + 460 \text{ R})(75 + 460 \text{ R}) - 1} = 26.75 \]

The cooling load of this air-conditioning system is the sum of the heat gain from the outside and the heat generated within the house,
\[ Q_L = 800 + 100 = 900 \text{ Btu/min} \]

The power input to this refrigerator is determined from the definition of the coefficient of performance of a refrigerator,
\[ W_{\text{net, in, min}} = \frac{Q_L}{\text{COP}_{R_{\text{max}}}} = \frac{900 \text{ Btu/min}}{26.75} = 33.6 \text{ Btu/min} = 0.79 \text{ hp} \]
7-103 A heat pump maintains a house at a specified temperature. The rate of heat loss of the house and the power consumption of the heat pump are given. It is to be determined if this heat pump can do the job.

**Assumptions** The heat pump operates steadily.

**Analysis** The power input to a heat pump will be a minimum when the heat pump operates in a reversible manner. The coefficient of performance of a reversible heat pump depends on the temperature limits in the cycle only, and is determined from

\[
\text{COP}_{\text{HP,rev}} = \frac{1}{1 - (T_r/T_H)} = \frac{1}{1 - (2 + 273 \text{ K})/(22 + 273 \text{ K})} = 14.75
\]

The required power input to this reversible heat pump is determined from the definition of the coefficient of performance to be

\[
W_{\text{net, min}} = \frac{Q_H}{\text{COP}_{\text{HP}}} = \frac{110,000 \text{ kJ h}}{14.75} \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) = 2.07 \text{ kW}
\]

This heat pump is **powerful enough** since 5 kW > 2.07 kW.

7-110 A Carnot heat pump consumes 8-kW of power when operating, and maintains a house at a specified temperature. The average rate of heat loss of the house in a particular day is given. The actual running time of the heat pump that day, the heating cost, and the cost if resistance heating is used instead are to be determined.

**Analysis** (a) The coefficient of performance of this Carnot heat pump depends on the temperature limits in the cycle only, and is determined from

\[
\text{COP}_{\text{HP,rev}} = \frac{1}{1 - (T_r/T_H)} = \frac{1}{1 - (2 + 273 \text{ K})/(20 + 273 \text{ K})} = 16.3
\]

The amount of heat the house lost that day is

\[Q_H = Q_H(1 \text{ day}) = (82,000 \text{ kJ h})(24 \text{ h}) = 1,968,000 \text{ kJ}\]

Then the required work input to this Carnot heat pump is determined from the definition of the coefficient of performance to be

\[
W_{\text{net, in}} = \frac{Q_H}{\text{COP}_{\text{HP}}} = \frac{1,968,000 \text{ kJ}}{16.3} = 120,736 \text{ kJ}
\]

Thus the length of time the heat pump ran that day is

\[\Delta t = \frac{W_{\text{net, in}}}{8 \text{ kJ/s}} = \frac{120,736 \text{ kJ}}{8 \text{ kJ/s}} = 15,092 \text{ s} = 4.19 \text{ h}\]

(b) The total heating cost that day is

Cost = \(W \times \text{price} = (W_{\text{net, in}} \times \Delta t)(\text{price}) = (8 \text{ kW})(4.19 \text{ h})(0.085 \$ / \text{kWh}) = 2.85\$

(c) If resistance heating were used, the entire heating load for that day would have to be met by electrical energy. Therefore, the heating system would consume 1,968,000 kJ of electricity that would cost

\[
\text{New Cost} = Q_H \times \text{price} = (1,968,000 \text{ kJ}) \left( \frac{1 \text{ kWh}}{3600 \text{ kJ}} \right) (0.085 \$ / \text{kWh}) = 46.47\$
\]
7-127 Two Carnot heat engines operate in series between specified temperature limits. If the thermal efficiencies of both engines are the same, the temperature of the intermediate medium between the two engines is to be determined.

**Assumptions** The engines are said to operate on the Carnot cycle, which is totally reversible.

**Analysis** The thermal efficiency of the two Carnot heat engines can be expressed as

\[ \eta_{\text{HE,1}} = 1 - \frac{T}{T_H} \quad \text{and} \quad \eta_{\text{HE,2}} = 1 - \frac{T_L}{T} \]

Equating,

\[ 1 - \frac{T}{T_H} = 1 - \frac{T_L}{T} \]

Solving for \( T \),

\[ T = \sqrt{T_H T_L} = \sqrt{(1800 \text{ K})(300 \text{ K})} = 735 \text{ K} \]

7-132 A Carnot heat engine drives a Carnot refrigerator that removes heat from a cold medium at a specified rate. The rate of heat supply to the heat engine and the total rate of heat rejection to the environment are to be determined.

**Analysis** (a) The coefficient of performance of the Carnot refrigerator is

\[ \text{COP}_{\text{R,C}} = \frac{1}{(T_H/T_L)-1} = \frac{1}{(300 \text{ K})/(258 \text{ K})-1} = 6.14 \]

Then power input to the refrigerator becomes

\[ W_{\text{in}} = \frac{Q_L}{\text{COP}_{\text{R,C}}} = \frac{400 \text{ kJ/min}}{6.14} = 65.1 \text{ kJ/min} \]

which is equal to the power output of the heat engine, \( W_{\text{out}} \).

The thermal efficiency of the Carnot heat engine is determined from

\[ \eta_{\text{HE,1}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ K}}{750 \text{ K}} = 0.60 \]

Then the rate of heat input to this heat engine is determined from the definition of thermal efficiency to be

\[ Q_{H,\text{HE}} = \frac{W_{\text{out}}}{\eta_{\text{HE,1}}} = \frac{65.1 \text{ kJ/min}}{0.60} = 108.5 \text{ kJ/min} \]

(b) The total rate of heat rejection to the ambient air is the sum of the heat rejected by the heat engine (\( Q_{L,HE} \)) and the heat discarded by the refrigerator (\( Q_{H,R} \)),

\[ Q_{L,HE} = Q_{H,HE} - W_{\text{in}} = 108.5 - 65.1 = 43.4 \text{ kJ/min} \]

\[ Q_{H,R} = Q_{L,R} + W_{\text{in}} = 400 + 65.1 = 465.1 \text{ kJ/min} \]

and

\[ Q_{\text{Ambient}} = Q_{L,HE} + Q_{H,R} = 43.4 + 465.1 = 508.5 \text{ kJ/min} \]