Problem 10-29E
A wall is constructed of two layers of 0.7-in-thick sheetrock \( k = 0.10 \text{ BTU/hr ft } ^\circ\text{F} \), which is a plasterboard made of two layers of heavy paper separated by a layer of gypson, placed 7 inches apart. The space between the sheetrocks is filled with fiberglass insulation \( k = 0.02 \text{ BTU/hr ft } ^\circ\text{F} \). Determine a) the thermal resistance of the wall, and b) its R-value of insulation in English units.

\[
R_{\text{sheetrock}} = R_1 = R_3 = \frac{L_1}{k_1} = \frac{0.7/12 \text{ ft}}{0.10 \text{ Btu/h.ft.}^\circ\text{F}} = 0.583 \text{ ft}^2 \cdot ^\circ\text{F.h/Btu}
\]

\[
R_{\text{fiberglass}} = R_2 = \frac{L_2}{k_2} = \frac{7/12 \text{ ft}}{0.020 \text{ Btu/h.ft.}^\circ\text{F}} = 29.17 \text{ ft}^2 \cdot ^\circ\text{F.h/Btu}
\]

\[
R_{\text{total}} = 2R_1 + R_2 = 2 \times 0.583 + 29.17 = 30.34 \text{ ft}^2 \cdot ^\circ\text{F.h/Btu}
\]

(b) Therefore, this is approximately a R-30 wall in English units.
Problem 10-30
The roof of a house consists of a 15-cm-thick concrete slab (k = 2 W/m °C) that is 15 m wide and 20 m long. The convective heat transfer coefficients on the inner and outer surfaces of the roof are 5 and 12 W/m²·°C. On a clear winter night, the ambient air is 10°C, while the night sky radiates at an effective temperature of 100°C. The house and the interior surfaces of the wall are maintained at a constant temperature of 20°C. The emissivity of both surfaces of the concrete roof is 0.9. Considering both radiation and convection heat transfer at the surfaces, determine a) the steady rate of heat transfer through the roof, and b) the inner surface temperature of the roof.

\[
\dot{Q} = \dot{Q}_{\text{room to roof, conv+rad}} + \dot{Q}_{\text{roof, cond}} = \dot{Q}_{\text{roof to surroundings}}
\]

\[
\dot{Q}_{\text{room to roof, conv+rad}} = h_A (T_{\text{room}} - T_{s,\text{in}}) + \varepsilon A \sigma (T_{\text{room}}^4 - T_{s,\text{in}}^4)
\]

\[
= (5 \text{ W/m}^2 \cdot \text{°C})(300 \text{ m}^2)(20 - T_{s,\text{in}}) \text{°C}
\]

\[
+ (0.9)(300 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)
\]

\[
(20 + 273 \text{ K})^4 - (T_{s,\text{in}} + 273 \text{ K})^4
\]

\[
\dot{Q}_{\text{roof, cond}} = kA \frac{T_{s,\text{in}} - T_{s,\text{out}}}{L} = (2 \text{ W/m} \cdot \text{°C})(300 \text{ m}^2) \frac{T_{s,\text{in}} - T_{s,\text{out}}}{0.15 \text{ m}}
\]

\[
\dot{Q}_{\text{roof to surr, conv+rad}} = h_o A (T_{s,\text{out}} - T_{\text{surr}}) + \varepsilon A \sigma (T_{s,\text{out}}^4 - T_{\text{surr}}^4)
\]

\[
= (12 \text{ W/m}^2 \cdot \text{°C})(300 \text{ m}^2)(T_{s,\text{out}} - 10) \text{°C}
\]

\[
+ (0.9)(300 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(T_{s,\text{out}} + 273 \text{ K})^4 - (100 \text{ K})^4]
\]

\[
\dot{Q} = 37,440 \text{ W}, T_{s,\text{in}} = 7.3 \text{°C}, \text{and } T_{s,\text{out}} = -2.1 \text{°C}
\]

The total amount of natural gas consumption during a 14-hour period is

\[
Q_{\text{gas}} = \frac{Q_{\text{total}}}{0.80} = \frac{\dot{Q} \Delta t}{0.80} = \frac{(37,440 \text{ kJ/s})(14 \times 3600 \text{ s})}{0.80} \left( \frac{1 \text{ therm}}{105,500 \text{ kJ}} \right) = 22.36 \text{ therms}
\]
Problem 10-38
Heat is conducted in parallel along a circuit board that has a copper layer on one side. The circuit board is 15 cm long (L) and 15 cm wide (w). The thicknesses of the copper and epoxy layers are 0.1 mm and 1.2 mm, respectively. Disregarding heat transfer from side surfaces, determine the percentages of heat conduction along the copper \((k = 386 \text{ W/m} \cdot \text{°C})\) and epoxy \((k = 0.26 \text{ W/m} \cdot \text{°C})\) layers. Also, determine the effective thermal conductivity \((k_{\text{eff}})\) of the board.

\[
\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} = \left( kA \frac{\Delta T}{L} \right)_{\text{copper}} + \left( kA \frac{\Delta T}{L} \right)_{\text{epoxy}}
\]

\[
= \left[ (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \right] v \frac{\Delta T}{L}
\]

\[
\dot{Q} = \left( kA \frac{\Delta T}{L} \right)_{\text{board}} = k_{\text{eff}} (t_{\text{copper}} + t_{\text{epoxy}}) w \frac{\Delta T}{L}
\]

\[
k_{\text{eff}} (t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}}
\]

\[
k_{\text{eff}} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}
\]

\[
(kt)_{\text{copper}} = (386 \text{ W/m} \cdot \text{°C})(0.0001 \text{ m}) = 0.0386 \text{ W/°C}
\]

\[
(kt)_{\text{epoxy}} = (0.26 \text{ W/m} \cdot \text{°C})(0.0012 \text{ m}) = 0.000312 \text{ W/°C}
\]

\[
(kt)_{\text{total}} = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} = 0.0386 + 0.000312 = 0.038912 \text{ W/°C}
\]

\[
f_{\text{epoxy}} = \frac{(kt)_{\text{epoxy}}}{(kt)_{\text{total}}} = \frac{0.000312}{0.038912} = 0.008 = 0.8\%
\]

\[
f_{\text{copper}} = \frac{(kt)_{\text{copper}}}{(kt)_{\text{total}}} = \frac{0.0386}{0.038912} = 0.992 = 99.2\%
\]

\[
k_{\text{eff}} = \frac{(386 \times 0.0001 + 0.26 \times 0.0012) \text{ W/°C}}{(0.0001 + 0.0012) \text{ m}} = 29.9 \text{ W/m} \cdot \text{°C}
\]
Problem 10-63
A 5-m-wide, 4-m-high, and 40-m-long kiln used to cure concrete pipes is made of 20-cm-thick concrete walls and ceiling (\( k = 0.9 \) W/m °C). The kiln is maintained at 40°C by injecting hot steam into it. The two ends of the kiln (4 m x 5 m in size) are made of 3-mm-thick sheet metal covered with 2-cm-thick styrofoam (\( k = 0.033 \) W/m °C). \( h_{\text{inner}} = 3000 \) W/m² °C and \( h_{\text{outer}} = 25 \) W/m² °C. Determine the rate of heat loss from the kiln when the ambient air is at -4°C. Assume the kiln is sitting on the floor with negligible heat transfer through the floor.

\[
T_{\text{out}} = -4°C
\]

\[
\begin{align*}
\text{Area}_{\text{inside}} &= 2 \times 40 \times (4 - 0.4) + 1 \times 40 \times (5 - 0.4) = 472 \text{ m}^2 \\
\text{Area}_{\text{outside}} &= 2 \times 40 \times 4 + 1 \times 40 \times 5 = 520 \text{ m}^2 \\
\end{align*}
\]

For convection:

\[
\text{Area}_{\text{concrete}} = 2 \times 40 \times (4 - 0.2) + 1 \times 40 \times (5 - 0.2) = 496 \text{ m}^2 \quad \text{(effective area)}
\]

\[
\text{Area}_{\text{end}} = (4 - 0.2) \times (5 - 0.2) = 18.24 \text{ m}^2 \quad \text{(average one side)}
\]
Problem 10-63 (Continued)
A 5-m-wide, 4-m-high, and 40-m-long kiln used to cure concrete pipes is made of 20-cm-thick concrete walls and ceiling (k = 0.9 W/m °C). The kiln is maintained at 40°C by injecting hot steam into it. The two ends of the kiln (4 m x 5 m in size) are made of 3-mm-thick sheet metal covered with 2-cm-thick styrofoam (k = 0.033 W/m °C). \( h_{\text{inner}} = 3000 \text{ W/m}^2 \text{ °C} \) and \( h_{\text{outer}} = 25 \text{ W/m}^2 \text{ °C} \). Determine the rate of heat loss from the kiln when the ambient air is at –4°C. Assume the kiln is sitting on the floor with negligible heat transfer through the floor.

\[
\begin{align*}
R_i &= \frac{1}{h_i A_i} = \frac{1}{(3000 \text{ W/m}^2 \cdot \text{°C})(472 \text{ m}^2)} = 0.0071 \times 10^{-3} \text{ °C/W} \\
R_{\text{concrete}} &= \frac{L}{k A_{\text{ave}}} = \frac{0.2 \text{ m}}{(0.9 \text{ W/m} \cdot \text{°C})(496 \text{ m}^2)} = 4.48 \times 10^{-4} \text{ °C/W} \\
R_o &= \frac{1}{h_o A_o} = \frac{1}{(25 \text{ W/m}^2 \cdot \text{°C})(520 \text{ m}^2)} = 0.769 \times 10^{-4} \text{ °C/W} \\
R_{\text{total}} &= R_i + R_{\text{concrete}} + R_o = (0.0071 + 4.48 + 0.769) \times 10^{-4} = 5.256 \times 10^{-4} \text{ °C/W} \\
\dot{Q}_{\text{top+sidess}} &= \frac{T_{\text{in}} - T_{\text{out}}}{R_{\text{total}}} = \frac{[40 - (-4)] \text{°C}}{5.256 \times 10^{-4} \text{ °C/W}} = 83,714 \text{ W}
\end{align*}
\]
Problem 10-63 (Continued)
A 5-m-wide, 4-m-high, and 40-m-long kiln used to cure concrete pipes is made of 20-cm-thick concrete walls and ceiling \((k = 0.9 \text{ W/m } \text{°C})\). The kiln is maintained at 40°C by injecting hot steam into it. The two ends of the kiln \((4 \text{ m x 5 m in size})\) are made of 3-mm-thick sheet metal covered with 2-cm-thick styrofoam \((k = 0.033 \text{ W/m } \text{°C})\). \(h_{\text{inner}} = 3000 \text{ W/m}^2 \text{°C}\) and \(h_{\text{outer}} = 25 \text{ W/m}^2 \text{°C}\). Determine the rate of heat loss from the kiln when the ambient air is at −4°C. Assume the kiln is sitting on the floor with negligible heat transfer through the floor.

\[
R_i = \frac{1}{h_i A_i} = \frac{1}{(3000 \text{ W/m}^2 \cdot \text{°C})[(4 - 0.4)(5 - 0.4) \text{ m}^2]} = 0.201 \times 10^{-4} \text{ °C/W}
\]

\[
R_{\text{sheet metal}} = \frac{L}{k A_{\text{avg}}} = \frac{3 \times 10^{-6} \text{ m}}{15 \text{ W/m} \cdot \text{°C} (18.24 \text{ m}^2)} = 1.09 \times 10^{-8} \text{ °C/W}
\]

\[
R_{\text{styrofoam}} = \frac{L}{k A_{\text{ave}}} = \frac{0.02 \text{ m}}{(0.033 \text{ W/m} \cdot \text{°C}) (18.24 \text{ m}^2)} = 0.0332 \text{ °C/W}
\]

\[
R_o = \frac{1}{h_o A_o} = \frac{1}{(25 \text{ W/m}^2 \cdot \text{°C})[4 \times 5 \text{ m}^2]} = 0.0020 \text{ °C/W}
\]

\[
R_{\text{total}} = R_i + R_{\text{styrofoam}} + R_o = 0.201 \times 10^{-4} + 0.0332 + 0.0020 = 0.0352 \text{ °C/W}
\]

and

\[
\dot{Q}_{\text{end surface}} = \frac{T_{\text{in}} - T_{\text{out}}}{R_{\text{total}}} = \frac{[40 - (-4)]\text{°C}}{0.0352 \text{ °C/W}} = 1250 \text{ W}
\]

Then the total rate of heat transfer from the kiln becomes

\[
\dot{Q}_{\text{total}} = \dot{Q}_{\text{top+ sides}} + 2 \dot{Q}_{\text{ends}} = 83,714 + 2(1250) = 86,214 \text{ W}
\]
Problem 10-72
Steam at 320°C flows in a stainless steel pipe (k = 15 W/m °C) whose inner and outer diameters are 5 cm and 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation (k = 0.038 W/m °C). Heat is lost to the surroundings at 5°C by convection and radiation with a combined heat transfer coefficient of 15 W/m²°C. \( h_{\text{inside}} = 80 \text{ W/m}^2\text{°C} \). Determine a) the rate of heat loss from the steam per unit length of the pipe, and b) the temperature drops across the pipe shell and insulation.

\[ A_i = \pi D_i L = \pi(0.05 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2 \]
\[ A_o = \pi D_o L = \pi(0.055 + 0.06 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2 \]

The individual thermal resistances are:

\[ R_i = \frac{1}{h_i A_i} = \frac{1}{(80 \text{ W/m}^2\cdot{°C})(0.157 \text{ m}^2)} = 0.08 \text{ °C/W} \]
\[ R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(15 \text{ W/m} \cdot °\text{C})(1 \text{ m})} = 0.00101 \text{ °C/W} \]
\[ R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.038 \text{ W/m} \cdot °\text{C})(1 \text{ m})} = 3.089 \text{ °C/W} \]
\[ R_o = \frac{1}{h_o A_o} = \frac{1}{(15 \text{ W/m}^2\cdot{°C})(0.361 \text{ m}^2)} = 0.1847 \text{ °C/W} \]
\[ R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.08 + 0.00101 + 3.089 + 0.1847 = 3.355 \text{ °C/W} \]

Then the steady rate of heat loss from the steam per m. pipe length becomes:
\[ \dot{Q} = \frac{T_{\text{wall}} - T_{\text{atmosphere}}}{R_{\text{total}}} = \frac{(320 - 5)\text{ °C}}{3.355 \text{ °C/W}} = 93.9 \text{ W} \]

The temperature drops across the pipe and the insulation are:
\[ \Delta T_{\text{pipe}} = \dot{Q} R_{\text{pipe}} = (93.9 \text{ W})(0.00101 \text{ °C/W}) = 0.095 \text{ °C} \]
\[ \Delta T_{\text{insulation}} = \dot{Q} R_{\text{insulation}} = (93.9 \text{ W})(3.089 \text{ °C/W}) = 290 \text{ °C} \]
A 10-m-long, 2.2-mm-diameter electric wire is tightly wrapped with a 1-mm-thick plastic cover whose thermal conductivity $k = 0.15 \text{ W/m} \cdot \text{°C}$. A current of 13 amps passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30 \text{°C}$ with a heat transfer coefficient $h = 24 \text{ W/m}^2 \cdot \text{°C}$, determine the steady state temperature at the interface of the wire and the plastic cover. Also, determine if doubling the thickness of the plastic will increase or decrease the interface temperature.

\[ \dot{Q} = \dot{W}_e = VI = (8 \text{ V})(13 \text{ A}) = 104 \text{ W} \]

The total thermal resistance is

\[ R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(24 \text{ W/m}^2 \cdot \text{°C})(\pi(0.0042 \text{ m})(10 \text{ m}))} = 0.3158 \text{°C/W} \]

\[ R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi nkL} = \frac{\ln(2.1/1.1)}{2\pi(0.15 \text{ W/m} \cdot \text{°C})(10 \text{ m})} = 0.0686 \text{°C/W} \]

\[ R_{\text{total}} = R_{\text{conv}} + R_{\text{plastic}} = 0.3158 + 0.0686 = 0.3844 \text{°C/W} \]

Then the interface temperature becomes

\[ \dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \implies T_1 = T_\infty + \dot{Q}R_{\text{total}} = 30 \text{°C} + (104 \text{ W})(0.3844 \text{°C/W}) = 70.0 \text{°C} \]

The critical radius of plastic insulation is

\[ r_{cr} = \frac{k}{h} = \frac{0.15 \text{ W/m} \cdot \text{°C}}{24 \text{ W/m}^2 \cdot \text{°C}} = 0.00625 \text{ m} = 6.25 \text{ mm} \]

Doubling the thickness of the plastic cover will increase the outer radius of the wire to 3 mm, which is less than the critical radius of insulation. Therefore, doubling the thickness of the plastic cover will increase the rate of heat loss, and decrease the interface temperature.
Problem 10-110

Obtain a relation for the fin efficiency of a fin with $A_c = \text{constant}$, perimeter $p$, length $L$ and thermal conductivity $k$ that is exposed to a medium at $T_\infty$ with convective heat transfer coefficient $h$. Assume that the fin is long, that the temperature at the end is $T_b$, and that there is no heat transfer at the end. $T_b$ is the temperature at the base. Simplify the relation for a) a circular fin of diameter $D$ and b) a rectangular fin of thickness $t$.

From Equation 10-70 (Page 436)

$$\eta_{\text{fin}} = \frac{\sqrt{hpkA_c}}{hA_{\text{fin}} (T_b - T_\infty)} = \frac{\sqrt{hpkA_c}}{hpL} = \frac{1}{L} \sqrt{\frac{kA_c}{\rho h}}$$

For a circular fin of diameter $D$:

$$\eta_{\text{fin, circular}} = \frac{1}{L} \sqrt{\frac{kA_c}{\rho h}} = \frac{1}{L} \sqrt{\frac{k}{4(\pi D)^2 h}} = \frac{1}{2L} \sqrt{\frac{kD}{h}}$$

For a rectangular fin of thickness $t$:

$$\eta_{\text{fin, rect}} = \frac{1}{L} \sqrt{\frac{kA_c}{\rho h}} = \frac{1}{L} \sqrt{\frac{k(wt)}{2(w+t)h}} = \frac{1}{L} \sqrt{\frac{kt}{2h}}$$
Problem 10-110
Obtain a relationship for the efficiency a fin of constant cross sectional area $A_c$, perimeter $p$, length $L$, and thermal conductivity $k$ that is exposed to a convection medium at $T_\infty$ with a heat transfer coefficient $h$. Assume the fins are long such that the temperature at the end is $T_\infty$. The temperature at the base is $T_b$. Simplify the relation for a) circular fin of diameter $D$ and b) rectangular fin at thickness $t$.

$$\eta_{\text{fin}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin}}$$

if the entire fin were at base temperature

$$= \frac{\sqrt{hpA_c (T_b - T_\infty)}}{hA_{\text{fin}} (T_b - T_\infty)} = \frac{\sqrt{hpA_c}}{hpL} = \frac{1}{L} \sqrt{\frac{kA_c}{ph}}$$

This relation can be simplified for a circular fin of diameter $D$ and rectangular fin of thickness $t$ and width $w$ to be

$$\eta_{\text{fin,circular}} = \frac{1}{L} \sqrt{\frac{kA_c}{ph}} = \frac{1}{L} \sqrt{\frac{k(\pi D^2/4)}{(\pi D)h}} = \frac{1}{2L} \sqrt{\frac{kD}{h}}$$

$$\eta_{\text{fin,rectangular}} = \frac{1}{L} \sqrt{\frac{kA_c}{ph}} = \frac{1}{L} \sqrt{\frac{k(wt)}{2(w+t)h}} = \frac{1}{L} \sqrt{\frac{k(wt)}{2wh}} = \frac{1}{L} \sqrt{\frac{kt}{2h}}$$
Problem 10-117

Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180°C. The aluminum fins (k = 186 W/m °C) are attached to the tube, have an outer diameter of 6 cm, and a constant thickness of 1 mm. There are 250 fins per meter length of tube each spaced 3 mm apart. Heat is transferred to the surrounding air at $T_\infty = 25$°C where $h = 40$ W/m²°C. Determine the increase heat transfer from the tube per meter as a result of adding fins.

$$A_{\text{no fin}} = \pi D_1 L = \pi(0.05 \text{ m})(1 \text{ m}) = 0.1571 \text{ m}^2$$

$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}} (T_b - T_\infty) = (40 \text{ W/m}^2\cdot\text{°C})(0.1571 \text{ m}^2)(180 - 25)\text{°C} = 974 \text{ W}$$

The efficiency of these circular fins is, from the efficiency curve, Fig. 10-43 180°C

$$L = (D_2 - D_1)/2 = (0.06 - 0.05)/2 = 0.005 \text{ m}$$

$$\frac{r_2 + (t/2)}{r_1} = \frac{0.03 + (0.001/2)}{0.025} = 1.22$$

$$\frac{L^{3/2}}{kA_p} = \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}}$$

$$= \left(0.005 + \frac{0.001}{2}\right)\sqrt{\frac{40 \text{ W/m}^2\cdot\text{°C}}{(186 \text{ W/m}^2\cdot\text{°C})(0.001 \text{ m})}} = 0.08$$

$$\eta_{\text{fin}} = 0.97$$

Heat transfer from a single fin is

$$A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi(0.03^2 - 0.025^2) + 2\pi(0.03)(0.001) = 0.001916 \text{ m}^2$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} \dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}} (T_b - T_\infty)$$

$$= 0.97(40 \text{ W/m}^2\cdot\text{°C})(0.001916 \text{ m}^2)(180 - 25)\text{°C} = 11.53 \text{ W}$$
Problem 10-117

Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180°C. The aluminum fins (k = 186 W/m °C) are attached to the tube, have an outer diameter of 6 cm, and a constant thickness of 1 mm. There are 250 fins per meter length of tube each spaced 3 mm apart. Heat is transferred to the surrounding air at \( T_\infty = 25^\circ C \) where h = 40 W/m²°C. Determine the increase heat transfer from the tube per meter as a result of adding fins.

Heat transfer from a single unfinned portion of the tube is

\[
A_{unfin} = \pi D_1 s = \pi (0.05 \text{ m})(0.003 \text{ m}) = 0.0004712 \text{ m}^2
\]

\[
\dot{Q}_{unfin} = h A_{unfin} (T_b - T_\infty) = (40 \text{ W/m}^2\cdot\text{°C})(0.0004712 \text{ m}^2)(180 - 25)\text{°C} = 2.92 \text{ W}
\]

There are 250 fins and thus 250 interfin spacings per meter length of the tube. The total heat transfer to the finned tube is then determined from

\[
\dot{Q}_{total, fin} = n (\dot{Q}_{fin} + \dot{Q}_{unfin}) = 250(11.53 + 2.92) = 3613 \text{ W}
\]

Therefore the increase in heat transfer from the tube per meter of its length as a result of the fins is

\[
\dot{Q}_{increase} = \dot{Q}_{total, fin} - \dot{Q}_{unfin} = 3613 - 974 = 2639 \text{ W}
\]
Problem 10-117
Steam in a heating system flows through tubes whose outer diameter is 5 cm and whose walls are maintained at a temperature of 180°F. Circular aluminum allow fins (k = 186 W/m °C) of outer diameter 6 cm and a thickness of 1 mm are attached to the tube. The space between the fins is 3 mm, and there are 250 fins per meter length of tube. Heat is transferred to the surrounding air at T_∞ = 25°F, with h = 40 W/m² °C. Determine the increase in heat transfer from the tube per meter length as a result of adding the fin.

**Analysis** In case of no fins, heat transfer from the tube per meter of its length is

\[ A_{\text{no fin}} = \pi D_1 L = \pi (0.05 \text{ m})(1 \text{ m}) = 0.1571 \text{ m}^2 \]

\[ \dot{Q}_{\text{no fin}} = hA_{\text{no fin}} (T_b - T_\infty) = (40 \text{ W/m}^2 \cdot \text{°C})(0.1571 \text{ m}^2)(180 - 25)\text{°C} = 974 \text{ W} \]

The efficiency of these circular fins is, from the efficiency curve, Fig. 10-43

\[ L = (D_2 - D_1)/2 = (0.06 - 0.05)/2 = 0.005 \text{ m} \]

\[ \frac{r_2 + (t/2)}{r_1} = \frac{0.03 + (0.001/2)}{0.025} = 1.22 \]

\[ L_c^{3/2} \left( \frac{h}{kA_p} \right)^{1/2} = \left( L + \frac{t}{2} \right) \sqrt{\frac{h}{kt}} \]

\[ = \left( 0.005 + \frac{0.001}{2} \right) \sqrt{\frac{40 \text{ W/m}^2 \cdot \text{°C}}{186 \text{ W/m} \cdot \text{°C}(0.001 \text{ m})}} = 0.08 \]

\[ A_{\text{fin}} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi(0.03^2 - 0.025^2) + 2\pi(0.03)(0.001) = 0.001916 \text{ m}^2 \]

\[ \dot{Q}_{\text{fin}} = \eta_{\text{fin}} \dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}} (T_b - T_\infty) \]

\[ = 0.97(40 \text{ W/m}^2 \cdot \text{°C})(0.001916 \text{ m}^2)(180 - 25)\text{°C} \]

\[ = 11.53 \text{ W} \]

\[ A_{\text{unfin}} = \pi D_1 s = \pi (0.05 \text{ m})(0.003 \text{ m}) = 0.0004712 \text{ m}^2 \]

\[ \dot{Q}_{\text{unfin}} = hA_{\text{unfin}} (T_b - T_\infty) = (40 \text{ W/m}^2 \cdot \text{°C})(0.0004712 \text{ m}^2)(180 - 25)\text{°C} = 2.92 \text{ W} \]

\[ \dot{Q}_{\text{total, fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 250(11.53 + 2.92) = 3613 \text{ W} \]

\[ \dot{Q}_{\text{increase}} = \dot{Q}_{\text{total, fin}} - \dot{Q}_{\text{no fin}} = 3613 - 974 = 2639 \text{ W} \]