This print-out should have 8 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

001 (part 1 of 2) 10.0 points

Given: \( V_{\text{sphere}} = \frac{4 \pi R^3}{3} \), and \( A_{\text{sphere}} = 4 \pi R^2 \).

Consider a sphere, which is an insulator, where charge is uniformly distributed throughout.

Consider a spherical Gaussian surface with radius \( R_2 \), which is concentric to the sphere with a radius \( R \).

\( Q \) is the total charge inside the sphere.

The total amount of flux flowing through the Gaussian surface is given by

1. \( \Phi = \frac{4 Q}{\epsilon_0} \).
2. \( \Phi = \frac{Q}{\epsilon_0} \).
3. \( \Phi = \frac{Q}{4 \epsilon_0} \).
4. \( \Phi = \frac{2 Q}{\epsilon_0} \).
5. \( \Phi = \frac{Q}{8 \epsilon_0} \). correct
6. \( \Phi = \frac{Q}{2 \epsilon_0} \).

Explanation:

Basic Concept: Gauss' Law.

Solution: For spherical symmetric case,

\[
\Phi = 4 \pi r^2 E = \frac{Q_{\text{encl}}}{\epsilon_0}.
\]

\[
= \frac{Q}{\epsilon_0} \left[ \frac{4 \pi}{3} \left( \frac{R}{2} \right)^3 \right] = \frac{Q}{8 \epsilon_0}.
\]

002 (part 2 of 2) 10.0 points

The magnitude of the electric field \( \| \vec{E} \| \) at \( \frac{R}{2} \) is given by

1. \( \| \vec{E} \| = \frac{2 k Q}{R^2} \).
2. \( \| \vec{E} \| = \frac{k Q^2}{R^2} \).
3. \( \| \vec{E} \| = \frac{k Q^2}{2 R^2} \).
4. \( \| \vec{E} \| = \frac{2 k Q^2}{R^2} \).
5. \( \| \vec{E} \| = \frac{k Q}{2 R^2} \). correct
6. \( \| \vec{E} \| = \frac{k Q}{R^2} \).

Explanation:

Gauss’s Law gives us

\[
4 \pi r^2 E = \frac{Q_{\text{encl}}}{\epsilon_0} = \frac{Q}{\epsilon_0} \left[ \frac{4 \pi}{3} \left( \frac{R}{2} \right)^3 \right] = \frac{Q}{8 \epsilon_0},
\]

\[
E = \frac{Q}{4 \pi \left( \frac{R}{2} \right)^2 8 \epsilon_0} = \frac{Q}{4 \pi \epsilon_0 2 R^2} = \frac{k Q}{2 R^2}.
\]
10.0 points

Two charges \( q_1 \) and \( q_2 \) are separated by a distance \( d \) and exert a force \( F \) on each other. What is the new force \( F' \), if charge 1 is increased to \( q_1' = 5q \), charge 2 is decreased to \( q_2' = \frac{q_2}{2} \), and the distance is decreased to \( d' = \frac{d}{2} \)? Choose one

1. \( F' = \frac{25}{2}F \)
2. \( F' = 10F \) correct
3. \( F' = 50F \)
4. \( F' = 25F \)
5. \( F' = 20F \)
6. \( F' = \frac{5}{2}F \)
7. \( F' = \frac{5}{4}F \)
8. \( F' = 5F \)
9. \( F' = 100F \)
10. \( F' = \frac{25}{4}F \)

Explanation:

\[
F' = \frac{k q_1' q_2'}{r'^2} = \frac{k (5q_1) \left( \frac{q_2}{2} \right)}{\left( \frac{d}{2} \right)^2} = 10 \frac{k q_1 q_2}{d^2} = 10F
\]

\( F' = 10F \).

10.0 points

Three charges are arranged as shown in the figure. The Coulomb constant is \( 8.98755 \times 10^9 \) N \( \cdot \) m\(^2\)/C\(^2\).

Find the magnitude of the electrostatic force on the charge at the origin. Correct answer: 2.09166 nN.

Explanation:

Let : \((x_0, y_0) = (0 \text{ m}, 0 \text{ m}),\) \(q_0 = -0.9 \text{ nC},\)

\((x_1, y_1) = (3.1 \text{ m}, 0 \text{ m}),\) \(q_1 = 1.9 \text{ nC},\)

\((x_2, y_2) = (0 \text{ m}, -1.8 \text{ m}),\) and \(q_2 = 0.54 \text{ nC}.\)

The force \( \vec{F}_{10} \) (in the \( \hat{i} \) direction, \(-\hat{i}\)) between charge \(-9 \times 10^{-10} \text{ C}\) and \(1.9 \times 10^{-9} \text{ C}\) is

\[
\vec{F}_{10} = +k_e \frac{q_0 q_1}{x_1^2} (-\hat{i})
\]

\[
= (-8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(-0.9 \text{ nC})(1.9 \text{ nC})}{(3.1 \text{ m})^2} \hat{i}
\]

\[
= (1.59924 \times 10^{-9} \text{ N}) \hat{i}.
\]

The plus sign means the force \( \vec{F}_{10} \) is towards the positive \( x \)-axis. The force \( \vec{F}_{20} \) (in the \( \hat{j} \) direction, \(+\hat{j}\)) between charge \(-9 \times 10^{-10} \text{ C}\) and \(5.4 \times 10^{-10} \text{ C}\) is

\[
\vec{F}_{20} = +k_e \frac{q_0 q_2}{y_2^2} (+\hat{j})
\]
= (8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \\
\times \frac{(-0.9 \text{ nC})(0.54 \text{ nC})}{(-1.8 \text{ m})^2} = (-1.34813 \times 10^{-9} \text{ N}) \hat{j}.

The minus sign means the force \( \vec{F}_{20} \) is towards the negative y-axis. The magnitude of the total force exerted on the charge \(-9 \times 10^{-10} \text{ C} \) is equal to

\[ \| \vec{F} \| = \sqrt{F_{10}^2 + F_{20}^2} \]
\[ = \left[ (1.59924 \times 10^{-9} \text{ N})^2 + (-1.34813 \times 10^{-9} \text{ N})^2 \right]^{\frac{1}{2}} \]
\[ = 2.09166 \times 10^{-9} \text{ N} \]
\[ = 2.09166 \text{ N}. \]

005 (part 2 of 2) 10.0 points
What is the angle \( \theta \) between the electrostatic force on the charge at the origin and the positive x-axis? Answer in degrees as an angle between \(-180^\circ \) and \(180^\circ \) measured from the positive x-axis, with counterclockwise positive.
Correct answer: \(-40.1303^\circ \).

Explanation:
The angle between \( \vec{F} \) and the negative y-axis is

\[ \theta = \arctan \left( \frac{-1.34813 \times 10^{-9} \text{ N}}{1.59924 \times 10^{-9} \text{ N}} \right) \]
\[ = -40.1303^\circ. \]

keywords:

006 10.0 points
A particle of mass 29 g and charge 19 \( \mu \text{C} \) is released from rest when it is 34 cm from a second particle of charge \(-18 \mu \text{C} \).
Determine the magnitude of the initial acceleration of the 29 g particle.
Correct answer: 916.873 m/s\(^2\).

Explanation:

Let: \( m = 29 \text{ g} \), 
\( q = 19 \mu \text{C} = 1.9 \times 10^{-5} \text{ C} \), 
\( d = 34 \text{ cm} = 0.34 \text{ m} \), 
\( Q = -18 \mu \text{C} = -1.8 \times 10^{-5} \text{ C} \), and 
k\(_e\) = 8.9875 \times 10^9.

The force exerted on the particle is

\[ F = k_e \frac{|q_1| |q_2|}{d^2} = ma \]
\[ \| \vec{a} \| = k_e \frac{|\vec{q}| \| \vec{Q} |}{m d^2} \]
\[ = k_e \frac{|1.9 \times 10^{-5} \text{ C} \| -1.8 \times 10^{-5} \text{ C}|}{(0.029 \text{ kg})(0.34 \text{ m}^2)} \]
\[ = \boxed{916.873 \text{ m/s}^2}. \]

007 10.0 points
Two electrons exert a force of repulsion of 1.23 N on each other.
How far apart are they?
Correct answer: \(1.36936 \times 10^{-14} \text{ m} \).

Explanation:

Let: \( F = 1.23 \text{ N} \).

The electric force is

\[ F = k_e \frac{q_1 q_2}{d^2} \]
\[ d^2 = k_e \frac{q_1^2}{F} \]
\[ d = \sqrt{k_e \frac{q_1^2}{F}} \]
\[ = \sqrt{\frac{(8.987 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)}{1.23 \text{ N}}} \times \sqrt{(1.602 \times 10^{-19} \text{ C})^2} \]
\[ = \boxed{1.36936 \times 10^{-14} \text{ m}}. \]

008 10.0 points
Consider a long, uniformly charged, cylindrical insulator of radius \( R \) with charge density 1.3 \( \mu \text{C/m}^3 \). (The volume of a cylinder with radius \( r \) and length \( \ell \) is \( V = \pi r^2 \ell \).)
The value of the Permittivity of free space is \(8.85419 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\).

What is the magnitude of the electric field inside the insulator at a distance 1.8 cm from the axis (1.8 cm < \(R\))?

Correct answer: 1321.41 N/C.

Explanation:

Let: \(r = 1.8 \text{ cm} = 0.018 \text{ m}\),
\[\rho = 1.3 \mu\text{C}/\text{m}^3\]
\[= 1.3 \times 10^{-6} \text{ C}/\text{m}^3, \quad \text{and}\]
\[\varepsilon_0 = 8.85419 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2.\]

Using Gauss’ law,
\[\Phi_s = \frac{Q_{\text{enc}}}{\varepsilon_0}\]
\[2 \pi r \ell E = \frac{\pi r^2 \ell \rho}{\varepsilon_0}.\]

Thus
\[E = \frac{\rho}{2 \varepsilon_0} r\]
\[= \frac{(1.3 \times 10^{-6} \text{ C}/\text{m}^3) (0.018 \text{ m})}{2 (8.85419 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)}\]
\[= 1321.41 \text{ N/C}.\]