A typical wing box with the cross-section as shown below. The contribution of the individual stringers located at the corners is neglected comparing to the overall torsion rigidity of the entire cross-section. Hence, this structure can be considered as a single cell closed section with a nonuniform wall thickness. The material constants used for the skin are $G$ (shear modulus) $= 28$ GPa and $\nu$ (Poisson's ratio) $= 0.3$. The section is twisted at an angle, $\theta = 5^\circ$/m ($0.087$ rad/m). Determine:

a) The required torque that causes the twist angle, $\theta = 5^\circ$/m  
b) Shear flow at the circular portion ($t_2$ segment) and at the $t_3$ segment, and the corresponding shear stress at the circular portion ($t_2$ segment) and at the $t_3$ segment, respectively.

c) Repeat part “a” and “b” but this time consider this cross-section
Solution:

a)

First we need to calculate the area,

\[ A_m = \frac{1}{2} \pi (0.6)^2 + (2.0)(1.2) + \frac{1}{2} (1.2) \frac{0.6}{\tan(30^\circ)} = 3.589m^2 \]

To calculate, \( a = \int \frac{ds}{t} \),

\[
 a_{10} = \frac{1}{2} \frac{\pi \cdot d}{t_2} + \frac{2(2.0)}{t_1} + \frac{2(1.2)}{t_4}
\]

\[
 a_{10} = \frac{1}{2} \frac{\pi \cdot (1.2)}{0.005} + \frac{2(2.0)}{0.0075} + \frac{2(1.2)}{0.0025} = 1870.3
\]

Writing the expression for the angular twist of the cell,

\[ 2G \theta = \frac{1}{A_m} [qa_{10}] \]

From this last equation we can calculate the shear flow “q”,

\[ q = \frac{A_m 2G \theta}{a_{10}} \]

Substituting,

\[ q = \left( \frac{3.589m^2}{1870.3} \right) \left( 2 \left( 28 \times 10^9 Pa \right) \left( 0.087 rad/m \right) \right) = 9.35 \times 10^6 N/m \]

Finally, equating the external torque to the internal resisting torque:

\[ T = 2qa_m \]

\[ T = 2 \left( 9.35 \times 10^6 N/m \right) \left( 3.589m^2 \right) = 67.11 \times 10^6 Nm \]
b) All the shear flows are the same,

\[ q_2 = q_3 = q = 9.35 \times 10^6 \text{ N/m} \]

To calculate the shear stress at the circular portion,

\[ \tau_2 = \frac{q_2}{t_2} = \frac{9.35 \times 10^6 \text{ N/m}}{0.005m} = 1870 \text{ MPa} \]

And in the \( t_3 \) segment,

\[ \tau_3 = \frac{q_3}{t_3} = \frac{9.35 \times 10^6 \text{ N/m}}{0.0075m} = 1247 \text{ MPa} \]

c) \[ A_m = \frac{1}{2} \pi (0.6)^2 + (2.0)(1.2) = 2.965m^2 \]

\[ a_{10} = \frac{1}{2} \frac{\pi \cdot d}{t_2} + \frac{2(2.0)}{t_1} + \frac{(1.2)}{t_4} \]

\[ a_{10} = \frac{1}{2} \frac{\pi \cdot (1.2)}{0.005} + \frac{2(2.0)}{0.0075} + \frac{(1.2)}{0.0025} = 1390.3 \]

Writing the expression for the angular twist of the cell,

\[ 2G \theta = \frac{1}{A_m} [qa_{10}] \]

From this last equation we can calculate the shear flow \( "q" \),

\[ q = \frac{A_m 2G \theta}{a_{10}} \]

Substituting,

\[ q = \frac{(2.965m^2)2(28 \times 10^9 \text{ Pa})(0.087 rad/m)}{1390.3} = 10.42 \times 10^6 \text{ N/m} \]

Finally, equating the external torque to the internal resisting torque:
\[ T = 2qA_m \]

\[ T = 2\left(10.42\times10^6 \frac{N}{m}\right)(2.965m^2) = 61.82\times10^6 Nm \]

All the shear flows are the same,

\[ q_2 = q_3 = q = 10.42\times10^6 \frac{N}{m} \]

To calculate the shear stress at the circular portion,

\[ \tau_2 = \frac{q_2}{t_2} = \frac{10.42\times10^6 \frac{N}{m}}{0.005m} = 2085MPa \]

And in the \( t_3 \) segment,

\[ \tau_3 = \frac{q_3}{t_3} = \frac{10.42\times10^6 \frac{N}{m}}{0.0075m} = 1390MPa \]