Problem 4-1

A beam having the cross-section shown in the figure is subjected to a bending moment of 1500 Nm in a vertical plane. Calculate the maximum direct stress due to bending stating the point at which it acts.

Solution:

The position of the centroid of the section may be found by taking moments of areas about some convenient point. Thus,

\[(120 \times 8 + 80 \times 8)z = 120 \times 8 \times 4 + 80 \times 8 \times 48\]

\[
\Rightarrow z = 21.6\text{mm}
\]

and

\[(120 \times 8 + 80 \times 8)\bar{y} = 80 \times 8 \times 4 + 120 \times 8 \times 24\]
giving

$$\bar{y} = 16 \text{mm}$$

The next step is to calculate the section properties referred to axes Cyz.

$$I_{yy} = \frac{120x(8)^3}{12} + 120x8x(17.6)^2 + \frac{8x(80)^3}{12} + 80x8x(26.4)^2 = 1.09 \times 10^6 \text{mm}^4$$

$$I_{xz} = \frac{8x(120)^3}{12} + 120x8x(8)^2 + \frac{80x(8)^3}{12} + 80x8x(12)^2 = 1.31 \times 10^6 \text{mm}^4$$

$$I_{yz} = 120x8x8x17.6 + 80x8x(-12)x(-26.4) = 0.34 \times 10^6 \text{mm}^4$$

Since $M_y=1500$ Nm and $M_z=0$ we have that,

$$\sigma_x = \frac{M_y(I_{zz}z - I_{yz}y)}{I_{yy}I_{zz} - I_{yz}^2} + \frac{M_z(I_{yy}y - I_{yz}z)}{I_{yy}I_{zz} - I_{yz}^2}$$

$$\sigma_x = \frac{1500000((1.31 \times 10^6)z - (0.34 \times 10^6)y)}{(1.09 \times 10^6)(1.31 \times 10^6) - (0.34 \times 10^6)^2}$$

$$\sigma_x = 1.5z - 0.39y$$

in which the units are N and mm.

By inspection we see that $\sigma_x$ will be a maximum at $F$ where $y = -8 \text{mm}$, $z = -66.4 \text{mm}$. Thus,

$$\sigma_{x,\text{max}} = -96 \text{N/mm}^2 \text{ (compressive)}$$

In some cases the maximum value cannot be obtained by inspection so that values of $\sigma_z$ at several points must be calculated.