Problem 4-4

Find the moments of inertia of a uniform beam of a thin-walled angle section as shown in the figure.

Part A)

Part B)
Solution:

Part A)

First, we need to calculate the centroid of the cross-section.

\[
\bar{y}_c = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{(1m/2)(0.1m^2) + (0m)(0.1m^2)}{0.1m^2 + 0.1m^2} = 0.25m
\]

\[
\bar{z}_c = \frac{\sum z_i A_i}{\sum A_i} = \frac{(0m)(0.1m^2) + (1m/2)(0.1m^2)}{0.1m^2 + 0.1m^2} = 0.25m
\]

The next step is to calculate the inertia of the cross-section.

\[
I_{yy} = \frac{(1m)(0.1m^3)}{12} + (0.1m^2)(0.25m)^2 + \frac{(0.1m)(0.1m^3)}{12} + (0.1m^2)(0.25m)^2 = 0.020833m^4
\]

we can neglect this term because 0.1^3 is very small

These terms came from the Parallel Axis Method

\[
I_{zz} = \frac{(0.1m)(1m)^3}{12} + (0.1m^2)(0.25m)^2 + \frac{(1m)(0.1m)^3}{12} + (0.1m^2)(0.25m)^2 = 0.020833m^4
\]

\[
I_{yz} = \int yz dA = \int yz dA_1 + \int yz dA_2 = (-0.00625m^4) + (-0.00625m^4) = -0.0125m^4
\]

where,

\[
\int_{A1} yz dA = \int_{z_{c_y}}^{z_{c_y}} y_{c_z} z_{tdz} = y_{c_z} \left( \frac{z^2}{2} \right)_{z_{c_z}}^{z_{c_z}} = (-0.25)(0.1m) \left( \frac{z^2}{2} \right)_{-0.25}^{0.75} = (-0.25)(0.1m) \left( \frac{0.75^2}{2} - \frac{(-0.25)^2}{2} \right) = -0.00625m^4
\]

\[
\int_{A2} yz dA = \int_{y_{c_z}}^{y_{c_z}} z_{c_y} y_{tdy} = z_{c_y} \left( \frac{y^2}{2} \right)_{y_{c_z}}^{y_{c_z}} = (-0.25)(0.1m) \left( \frac{y^2}{2} \right)_{-0.25}^{0.75} = (-0.25)(0.1m) \left( \frac{0.75^2}{2} - \frac{(-0.25)^2}{2} \right) = -0.00625m^4
\]
Another way to calculate Iyz is,

\[ I_{yx} = \bar{I}_{yx} + A\bar{y}dz = [0 + (0.1m^2)(-0.25m)(0.25m)] + [0 + (0.1m^2)(0.25m)(-0.25m)] = -0.0125m^4 \]

Part B)

Now, we need to rotate the cross-section 45° counterclockwise.

\[
\begin{align*}
I_{y'y'} &= I_{yx}\cos^2(\theta) + I_{zx}\sin^2(\theta) + 2I_{yx}\sin(\theta)\cos(\theta) \\
I_{z'z'} &= I_{yx}\sin^2(\theta) + I_{zx}\cos^2(\theta) - 2I_{yx}\sin(\theta)\cos(\theta) \\
I_{y'z'} &= I_{yx}\sin(\theta)\cos(\theta) - I_{zx}\sin(\theta)\cos(\theta) + I_{yx}[\cos^2(\theta) - \sin^2(\theta)]
\end{align*}
\]

Substituting,

\[ I_{y'y'} = I_{yx}\cos^2(45°) + I_{zx}\sin^2(45°) + 2I_{yx}\sin(45°)\cos(45°) \]

\[ I_{y'y'} = (0.020833m^4)\left(\frac{1}{\sqrt{2}}\right)^2 + (0.020833m^4)\left(\frac{1}{\sqrt{2}}\right)^2 + 2(-0.0125m^4)\left(\frac{1}{\sqrt{2}}\right)^2 \]

\[ I_{y'y'} = 0.008333m^4 \]

\[ I_{z'z'} = I_{yx}\sin^2(45°) + I_{zx}\cos^2(45°) - 2I_{yx}\sin(45°)\cos(45°) \]

\[ I_{z'z'} = (0.020833m^4)\left(\frac{1}{\sqrt{2}}\right)^2 + (0.020833m^4)\left(\frac{1}{\sqrt{2}}\right)^2 - 2(-0.0125m^4)\left(\frac{1}{\sqrt{2}}\right)^2 \]

\[ I_{z'z'} = 0.03333m^4 \]

\[ I_{y'z'} = I_{yx}\sin(45°)\cos(45°) - I_{zx}\sin(45°)\cos(45°) + I_{yx}[\cos^2(45°) - \sin^2(45°)] \]

\[ I_{y'z'} = (0.020833m^4)\left(\frac{1}{\sqrt{2}}\right)^2 - (0.020833m^4)\left(\frac{1}{\sqrt{2}}\right)^2 \]

\[ I_{y'z'} = 0m^4 \]