Problem 8-1

A straight round steel shaft is subjected to a torque of 6,000 lb-in. The material has yielding strength of 60 Ksi and a safety factor of 2. Determine the minimum required diameter that will not give a yielding failure by using.

a) Maximum principal stress criterion
b) Maximum shear stress criterion
c) Von-Mises failure criterion

Solution:

For a round shaft under torque,

$$\tau_{MAX} = \frac{T}{J} = \frac{T}{ar^4} = \frac{2T}{ar^3} = \frac{12,000}{\pi} \frac{1}{r^3}$$

a) Maximum principal stress criterion

First we need to find the principal stresses. At the surface of the shaft, there is pure shear acting,

$$[\sigma] = \begin{bmatrix} 0 & \tau_{MAX} \\ \tau_{MAX} & 0 \end{bmatrix}$$

$$\sigma^2 - \tau_{MAX}^2 = 0$$

$$\sigma = \pm \sqrt{\tau_{MAX}^2}$$

$$\sigma_{1,2} = \pm \frac{12,000}{\pi} \frac{1}{r^3}$$

$$\sigma_1 = +\frac{12,000}{\pi} \frac{1}{r^3} \quad \text{and} \quad \sigma_2 = -\frac{12,000}{\pi} \frac{1}{r^3}$$

On the other hand,

$$S.F. = \frac{\sigma_{allow}}{\sigma_1} \rightarrow 2\sigma_1 = \sigma_{allow} \rightarrow \sigma_{allow} = 2 \left(\frac{12,000}{\pi} \frac{1}{r^3}\right) \rightarrow \sigma_{allow} = \frac{24,000}{\pi} \frac{1}{r^3}$$
\[ S.F. = \frac{\sigma_{2}^{\text{allow}}}{\sigma_{2}} \rightarrow 2\sigma_{2} = \sigma_{2}^{\text{allow}} \rightarrow \sigma_{2}^{\text{allow}} = 2 \left( \frac{12,000}{\pi} \frac{1}{r^3} \right) \rightarrow \sigma_{2}^{\text{allow}} = \frac{24,000}{\pi} \frac{1}{r^3} \]

Finally, when,

\[ \left| \sigma_{1}^{\text{allow}} \right| \leq \sigma_{y} \rightarrow \frac{24,000}{\pi} \frac{1}{r^3} = 60,000 \text{ psi} \rightarrow r = 0.50in \rightarrow d_{\text{min}} = 1.01in \]

\[ \left| \sigma_{2}^{\text{allow}} \right| \leq \sigma_{y} \rightarrow \frac{24,000}{\pi} \frac{1}{r^3} = 60,000 \text{ psi} \rightarrow r = 0.50in \rightarrow d_{\text{min}} = 1.01in \]

b) Maximum shear stress criterion

There are 3 conditions,

\[ \left| \sigma_{1}^{\text{allow}} - \sigma_{2}^{\text{allow}} \right| \leq \sigma_{y} \]
\[ \left| \sigma_{1}^{\text{allow}} \right| \leq \sigma_{y} \]
\[ \left| \sigma_{1}^{\text{allow}} \right| \leq \sigma_{y} \]

For the last two conditions we already know the minimum diameter possible to prevent yielding is \( d = 1.01 \text{ inches} \). However, for the first condition,

\[ \left( \frac{24,000}{\pi} \frac{1}{r^3} \right) - \left( -\frac{24,000}{\pi} \frac{1}{r^3} \right) = 60,000 \text{ psi} \]

\[ \frac{48,000}{\pi} \frac{1}{r^3} = 60,000 \text{ psi} \]

\[ r = \sqrt[3]{\frac{48,000}{60,000} \pi} \]

\[ r = 0.63in \]

\[ d_{\text{min}} = 1.27in \]

Therefore, the minimum diameter for the shaft has to be 1.27 inches.
c) Von-Mises failure criterion

\[(\sigma_1^{allow})^2 + (\sigma_2^{allow})^2 - (\sigma_1^{allow})(\sigma_2^{allow}) \leq (\sigma_f)^2\]

\[\left(\frac{24,000}{\pi r^3}\right)^2 + \left(-\frac{24,000}{\pi r^3}\right)^2 - \left(\frac{24,000}{\pi r^3}\right)\left(-\frac{24,000}{\pi r^3}\right) = (60,000)^2\]

\[3\left(\frac{24,000}{\pi r^3}\right)^2 = (60,000)^2\]

\[r = 0.60\text{in}\]

\[d_{\text{min}} = 1.21\text{in}\]
Problem 8-2
A large thin steel plate is loaded as shown below in each case. The material properties of the plate is given as

\[ E = 30 \times 10^6 \text{ psi}, \quad v = 0.3, \quad \sigma_{ult} = 82 \text{ ksi} \]

\[ \sigma_y = 47 \text{ ksi}, \quad K_c = 110 \text{ ksi}\sqrt{\text{in}} \]

An infinite thin plate has a 4” crack at the center. Determine:

(a) the max. permissible load under this crack length
(b) the max. permissible crack length if the plate is subjected to 50 ksi.

Solution:

Part a)

\[ K_c = \sigma_y \sqrt{\pi a} \]

\[ 110\text{ksi}\sqrt{\text{in}} = \sigma_y \sqrt{\pi 2\text{in}} \]

\[ \sigma_y = \frac{110\text{ksi}\sqrt{\text{in}}}{\sqrt{\pi 2\text{in}}} = 43.88\text{ksi} \]

Part b)

\[ K_c = \sigma_y \sqrt{\pi a_e} \]

\[ 110\text{ksi}\sqrt{\text{in}} = 50\text{ksi}\sqrt{\pi a_e} \]

\[ a_e = \left( \frac{110\text{ksi}\sqrt{\text{in}}}{50\text{ksi}\sqrt{\pi}} \right)^2 = 1.54\text{in} \]

Crack length = 3.08\text{in}