There are 5 numbered, equally weighted problems on this exam.
Time allowed for the exam is 80 minutes.
Read the problems carefully and don’t waste time doing work that is not requested.
Problem statements may include information that you do not really need.
Provided values have at least three significant digits even when not expressed that way.
**Draw Free Body Diagrams (or other diagrams) when appropriate!!!!**
Show each problem solution in detail. Test credit will be based primarily on your solution method rather than your numerical solution.
If you are filling each page, you are probably doing too much. If you do need additional space for a solution or scratch paper, use the back of the preceding test page.

**Formulas**

\[
\tau = \frac{T c}{J} \\
\phi = \int_0^x \frac{T(x)}{J} \, dx \\
\phi = \sum \frac{TL}{J} \\
\phi = \frac{TL}{J} \\
\sigma = \frac{P}{A} \\
\sigma = \frac{n}{E_2} \\
\sigma = n \sigma' \\
\tau = \frac{VQ}{It} \\
\sigma_1 = \frac{pr}{t} \\
\sigma_2 = \frac{pr}{2t} \\
\sigma = -\frac{M_y y + M_z z}{I_x} \\
Q = \nabla A' \\
\vec{M} = \vec{r} \times \vec{F} \\
\vec{F} = \begin{bmatrix}
\hat{i} & \hat{j} & \hat{k} \\
F_x & F_y & F_z \\
\end{bmatrix} \\
A = bh \\
I_x = \frac{bh^3}{12} \\
I_y = \frac{bh^3}{12} \\
A = \pi r^2 \\
I_x = \frac{\pi r^4}{4} \\
J = \frac{\pi r^4}{2} \\
A = \frac{\pi r^2}{2} \\
I_y = \frac{\pi r^4}{8} \\
\bar{x} = \frac{4r}{3\pi}
1) A cantilever beam supports a triangular distributed load as shown below on the left. At a later time, the same cantilever beam supports a triangular distributed load as shown below on the right.

a) Sketch the shear diagram (only) for each loading state. Provide the corresponding maximum value of shear in each sketch.

![Shear Diagrams](image)

b) The cross section of the beam is given below. Determine the shear stress at point C in the cross section at the fixed support, B, for the beam shear caused by the load case on the left.

![Cross Section](image)
2) The simply supported beam shown is supporting a concentrated load $P = 80$ kip at point B and an applied moment $M = 480$ kip-ft at point C. The resulting reactions are $A_y = 64$ kip and $D_y = 16$ kip.

Draw the shear and moment diagrams for the beam.
3) A beam is made from two different materials A and B. $E_A = 400$ ksi and $E_B = 800$ ksi. The dimensions of a cross section of the beam are shown at the right. A particular cross section of the beam is subjected to a bending moment $M = 1.10$ kip-ft as shown in the other diagram.

**Determine the maximum bending stress in material B.**

$$
\sigma = n \sigma' \\
\sigma = 2 \cdot \sigma' \\
\sigma = -222.972 \text{ psi}
$$

$$
\sigma' = \frac{-M_y y}{I_z} \\
\sigma' = \frac{-(-13.2 \cdot 10^3)(-5)}{592} \\
\sigma' = -111.486 \text{ psi}
$$

```latex
\begin{array}{cccc}
\bar{y} & A & \bar{y}A & I_o & Ad^2 \\
A & 8 & 24 & 192 & 128 & 216 \\
B' & 2 & 24 & 48 & 32 & 216 \\
Total & 48 & 240 & 160 & 432 \\
\end{array}
```

$$
\bar{y} = \frac{\sum \bar{y}A}{\sum A} \\
\bar{y} = \frac{240}{48} \\
\bar{y} = 5 \text{ in} \\
I_z = \sum I_o + \sum Ad^2 \\
I_z = 160 + 432 \\
I_z = 592 \text{ in}^2$

$$
\sigma = n \sigma' \\
\sigma = 2 \cdot \sigma' \\
\sigma = -223 \text{ psi}
$$
4) The circular steel rod shown has a diameter $D = 2$ in. The rod is fixed at A and C (reactions are shown) and is subjected to a torque at point B of $T_B = 9$ kip-in.

$L_{AB} = 15$ in $\quad L_{BC} = 12$ in

For steel, $E = 29000$ ksi, $G = 11000$ ksi

Determine the reactions at A and C.

\[
\begin{align*}
\sum M_x &= 0 \\
T_A + T_{AB} &= 0 \\
T_{AB} &= -T_A \\
\end{align*}
\]

\[
\begin{align*}
\sum M_x &= 0 \\
T_A + T_B + T_{BC} &= 0 \\
T_{BC} &= -T_A - T_B \\
\end{align*}
\]

\[
\phi_{AB} + \phi_{BC} = 0
\]

\[
\frac{T_{AB} L_{AB}}{J_{AB} G} + \frac{T_{BC} L_{BC}}{J_{BC} G} = 0
\]

\[
\begin{align*}
(-T_A) L_{AB} + (-T_A - T_B) L_{BC} &= 0 \\
(-T_A) 15 + (-T_A - 9) 12 &= 0 \\
T_A &= -4.00 \text{ kip-in} \\
\end{align*}
\]

From the given diagram

\[
\begin{align*}
\sum M_x &= 0 \\
T_A + T_B + T_C &= 0 \\
(-4) + 9 + T_C &= 0 \\
T_C &= -5.00 \text{ kip-in} \\
\end{align*}
\]
5) The 2 in diameter rod is fixed at the end containing point A. The centroidal axis of the longer part of the rod lies on the x axis. The rod is subjected to the load $P$ applied at point B. The force $P$ causes reactions at the support, $\vec{R}_i = (-120\hat{i} + 160\hat{j})$ lb, $\vec{M} = (-640\hat{i} - 480\hat{j} + 1920\hat{k})$ lb-in A is the point where the z axis passes through the surface of the rod ($r_z = (1\hat{k})$ in).

a) Determine the normal stress at point A due to the axial force in the rod

$$\sigma_p = \frac{P}{A}$$
$$\sigma_p = \frac{120}{\pi \cdot 1^2}$$
$$\sigma_p = 38.197$$

$$\sigma_p = 38.2 \text{ psi}$$

b) Determine the normal stress at point A due to the bending moment about the y- axis.

$$I_y = \frac{\pi r^4}{4}$$
$$I_y = \frac{\pi 1^4}{4}$$
$$I_y = 0.78540 \text{ in}^4$$

$$\sigma_{M_y} = \frac{M_y z}{I_y}$$
$$\sigma_{M_y} = \frac{480 \cdot 1}{0.78540}$$
$$\sigma_{M_y} = 611.15$$

$$\sigma_{M_y} = 611 \text{ psi}$$

c) Determine the normal stress at point A due to the bending moment about the z- axis.

$$I_z = \frac{\pi r^4}{4}$$
$$I_z = \frac{\pi 1^4}{4}$$
$$I_z = 0.78540 \text{ in}^4$$

$$\sigma_{M_z} = -\frac{M_z y}{I_z}$$
$$\sigma_{M_z} = -\frac{(-1920) \cdot 0}{0.78540}$$
$$\sigma_{M_z} = 0$$

$$\sigma_{M_z} = 0 \text{ psi}$$

d) Determine the total normal stress at point A.

$$\sigma = \sigma_p + \sigma_{M_y} + \sigma_{M_z}$$
$$\sigma = 38.197 + 611.15 + 0$$

$$\sigma = 649 \text{ psi}$$