For Stress Relaxation

from Maxwell model:

\[
\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} + \frac{1}{E} \frac{d\sigma}{dt}
\]

boundary conditions for stress relaxation:

\[\varepsilon = \varepsilon_0, \quad \frac{d\varepsilon}{dt} = 0\]

\(\sigma_0\) corresponds to initial stress when \(\varepsilon\) applied \(\sigma\) at \(t = 0\)

Thus,

\[0 = \frac{\sigma}{\eta} + \frac{1}{E} \frac{d\sigma}{dt}\]

rearranging,

\[
\frac{d\sigma}{\sigma} = d\ln\sigma = -\frac{E}{\eta} \frac{dt}{t}
\]
integrating from \( \sigma_0 @ t = 0 \) to \( \sigma(t) \) at time \( t \):

\[
\ln \sigma(t) = \ln \sigma_0 - \frac{Et}{\eta}
\]

exponentiation yields:

\[
\sigma(t) = \sigma_0 e^{-\frac{Et}{\eta}}
\]

or letting \( F = \frac{\eta}{E} \) and substituting:

\[
\sigma(t) = \sigma_0 e^{-t/F}
\]

where \( F \) is defined as the relaxation time.

\[
\frac{\sigma(t)}{\sigma_0} \rightarrow 1 \quad \text{as} \quad t \rightarrow \infty
\]
For Creep:

recalling Voigt equation:

\[ \sigma_T(t) = E \varepsilon + \eta \frac{d\varepsilon}{dt} \]

for a creep test, \( \sigma_T(t) = \sigma_0 \), therefore:

\[ \varepsilon(t) = \frac{\sigma_0}{E} \left(1 - e^{-t/\tau}\right) \]
recalling the Maxwell equation:

$$\frac{d\varepsilon}{dt} = -\frac{\sigma}{\eta} + \frac{1}{E} \frac{d\sigma}{dt}$$

or:

$$\frac{d\sigma}{dt} = E \frac{d\varepsilon}{dt} - \frac{E\sigma}{\eta}$$

for creep $d\sigma/dt = 0$; therefore:

$$\varepsilon = \varepsilon_0 + \frac{\sigma}{\eta}$$

$$\varepsilon = \varepsilon_0 \left(1 + \frac{t}{T}\right)$$
SERIES COMBINATIONS OF MAXWELL AND VOIGT MODELS

The total strain for creep conditions will be due to:

- an instantaneous elastic deformation (Maxwell spring element)
- an irrecoverable viscous flow (Maxwell element dashpot)
- a recoverable retarded elastic deformation (Voigt element)
Molecular mechanisms associated:

1. **Instantaneous elastic deformation** - bending and stretching of primary valence bonds.

2. **Irrecoverable viscous flow** - slippage of the polymer chain or chain segments past one another.

3. **Retarded elastic deformation** - the transformation of a given equilibrium conformation into a biased conformation in which elongated and oriented structures are favored.
For constant load:

\[ \epsilon = \frac{\sigma}{E_m} + \frac{\sigma t}{\eta_m} + \frac{\sigma}{E_v} \left(1 - e^{-t/t_v}\right) \]
Material Response Time

\[
\frac{6}{60} \quad T_1 \quad T_2 \quad T_3
\]

\[\text{time}\]

for Maxwell model

\[T_1 < T_2 < T_3\]

as \(T \to 0\); rapid decay; liquid-like behavior; completely viscous

as \(T \to \infty\); maintenance of high stress for long time; solid-like behavior; completely elastic

\[\tau = f(\eta) \quad \eta = f(T) \quad \Rightarrow \quad \text{low @ high } T \to \text{fluid-like}\]

\[\tau \text{ high @ low } T \to \text{solid-like}\]
Figure 5.8: The behaviour of the Maxwell and Voigt models during different types of loading.