1. (10 points) A particle at a 6.5-in radius is on a body that is in pure rotation with \( \omega = 100 \text{ rad/sec} \) CCW and a constant \( \alpha = -500 \text{ rad/sec}^2 \) at point A. The rotation center is at the origin of a coordinate system. When the particle is at position A, its position vector makes a \( 45^\circ \) angle with the X axis.

Write an expression for particle's acceleration vector in position A using complex number notation, in both polar and Cartesian forms. Show the acceleration vector in the drawing.

(Note: Euler identity: \( e^{2j\theta} = \cos \theta \pm j \sin \theta \))

**Analysis**

\[
\vec{r}_A = r_A e^{j \theta_A} \quad \alpha = A
\]

\[
\vec{v}_A = \frac{d\vec{r}_A}{dt} = \frac{d}{dt} (r_A e^{j \theta_A}) = r_A \frac{d}{dt} (e^{j \theta_A})
\]

\[
= r_A \frac{d}{dt} (\cos \theta_A + j \sin \theta_A)
\]

\[
= r_A (\frac{d}{dt} (\cos \theta_A + j \sin \theta_A)) = r_A (-\sin \theta_A + j \cos \theta_A)
\]

**Polar**

\[
\vec{a}_A = R \alpha e^{j \theta_A} = R \alpha \omega_A^2 e^{j \theta_A}
\]

\[
= R \alpha e^{j \theta_A} (\alpha - \omega_A^2)
\]

\[
= 6.5 j e^{\pi/4} \left(100^2 - 500^2 \right)
\]

\[
= \left(6.175 \times 10^4\right) j e^{0.25 \pi j} \text{ (in l/s)}
\]
Problem 1 (cont)

**Cartesian Form:**

\[ \vec{A}_A = R_A \vec{a}_A (-\sin \theta_A + j \cos \theta_A) \]

\[ = R_A \omega_A^2 (\cos \theta_A + j \sin \theta_A) \]

\[ = R_A \left[ \vec{a}_A (j \cos \theta_A - \sin \theta_A) - \omega_A^2 (\cos \theta_A + j \sin \theta_A) \right] \]

\[ = 6.5 \left[ (-500) (j \cos 45 - \sin 45) - 100^2 (\cos 45 + j \sin 45) \right] \]

\[ = -31380 \times 10^3 - 57020 \times 10^3 j \text{ in/} \text{s} \]

\[ -2 \]
2. (10 points) The rod AB is attached to a slider C at point A. The slider moves with a constant velocity of $V_c = 0.5 \text{ m/sec}$ to the right. Simultaneously, the rod AB is made to rotate about the pivot at A at a constant rate of 20 RPM. Determine the magnitude of the acceleration of point A on rod AB.

a. 20 RPM
b. 0.885 m/s²
c. 0.0 m/s²
d. 0.200 m/s²

3. (10 points) The rod AB is attached to a slider C at point A. The slider moves with a constant velocity of $V_c = 0.5 \text{ m/sec}$ to the right. Simultaneously, the rod AB is made to rotate about the pivot at A at a constant rate of 20 RPM. Determine the direction of the acceleration of point B on rod AB with respect to the x-axis.

a. 0°
b. 30°
c. 60°
d. 90°
- Make assumptions.
- Introduce as many variables as possible.

4. (15 points) Draw the free-body diagram of the coupler and write its dynamic equation with respect to a local Cartesian coordinate system at its CG with its x-axis parallel to the ground link. The mass of the couple is \( m \).

\[
\begin{align*}
\Sigma F &= m \ddot{a} \\
\Sigma T &= I \alpha
\end{align*}
\]

where:
- \( \Sigma F \) = Force Sum
- \( \Sigma T \) = Torque Sum

\( m \) = mass
\( a \) = acceleration
\( I \) = inertia
\( \alpha \) = any acceleration

At point C where \( AB \) lines intersect

At Point C is the CG point of coupler

At point B:

- Assuming tensile force \( CA \) \( \Rightarrow \) \( AB \)

Equation of Motion:

\[
\Sigma F = \Sigma (m \ddot{a})
\]

\[
\frac{F_A}{F_B} + \frac{F_B}{F_A} - F = m \ddot{a}
\]

Component Form :

\[
\begin{align*}
\Sigma F_x &= (F_A + F_B - F)_x = m a_x \\
\Sigma F_y &= (F_A + F_B - F)_y = m a_y \\
\Sigma T_x &= I_x \alpha \\
\Sigma T_y &= I_y \alpha
\end{align*}
\]

Component Form

\[
\begin{align*}
(F_A \bar{r}_{CA}) + (F_B \bar{r}_{CB}) - \bar{F} &= \bar{I} \alpha
\end{align*}
\]
5. (30 points) Suppose the linkage moves very slowly and links 2, 3 and 5 are all axial-force members. In the position shown the force of link 2 is given as $F_{O_2A} = 100$ lb (in compression). The link 5 is under compression as well. Answer the following questions.

a. Is link 3 under tension or compression? Why?

b. Determine force in link 5, $F_{AB}$

c. Determine force in link 3, $F_{AC}$

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**Measurements**

$O_2B = 5.5$ cm

$O_2A = 1.5$ cm

$AB = 4.3$ cm

$O_2C = 4.4$ cm

$AC = 3.8$ cm

---

"Quasi-stationary motion of overall mechanism behavior."

(a) Link 3 is under **Tension** because it connects with roller C which provides tensile motion pulling it away from point A.

(b) $\frac{F_{O_2A}}{O_2A} = \frac{F_{AB}}{AB} \Rightarrow F_{AB} = \left(\frac{F_{O_2A} \cdot AB}{O_2A}\right) = \frac{(100)(4.3)}{1.5}

\[ F_{AB} = 286.67 \text{ (lb)} \]

(c) $\frac{F_{AC}}{AC} = \frac{F_{AB}}{AB} \Rightarrow F_{AC} = \left(\frac{F_{AB} \cdot AC}{AB}\right) = \frac{(286.67)(3.8)}{4.3}

\[ F_{AC} = 253.33 \text{ (lb)} \]