1. The steady-state temperature distribution in a one-dimensional wall of thermal conductivity 100 W/m·K and thickness 1 cm is observed to be $T(0) = a + bx^2$, where $a = 200^\circ C$, $b = -2000^\circ C/m^2$, and $x$ is in meters.

(a) What is the heat generation rate per unit volume ($q$) in the wall? (1.0 pts)
(b) Determine the heat fluxes at the two wall faces. (1.0 pt)

For one-dimensional conduction

Energy equation

$\varphi (kT) + \frac{q}{\rho C_p} = \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right)$

$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{q}{\rho C_p} = 0$  (2.19 or 2.80)

$\frac{\partial T}{\partial x} = 2bx$

$\frac{\partial^2 T}{\partial x^2} = 2b$

$\therefore 2b + \frac{q}{\rho C_p} = 0 \Rightarrow \frac{q}{\rho C_p} = -2bk$

$= (\frac{-2}{100 W/m·K}) \cdot (-2000^\circ C/m^2)$

$= 400 \times 10^3 W/m^3 = 4.0 \times 10^5 W/m^3$

(b) $q''(x) = -k \frac{dT}{dx} = -2k bx$

$\begin{cases} 
q''(x=0) = 0 \\
q''(x=1 cm) = \frac{-2.(100 W/m·K).(-2000^\circ C/m^2).0.01 m}{400 \times 10^3 W/m^3}
\end{cases}$
2. Steam at 250°F flows in an insulated pipe. The pipe is 1% carbon steel \((k_{\text{steel}}=25 \text{ Btu/hr-ft-}^\circ\text{F})\) and has an inside radius of 2.0 in and outside radius of 2.25 in. The pipe is covered with a one-inch layer of 85% magnesia \((k_{\text{mag}}=0.041 \text{ Btu/hr-ft-}^\circ\text{F})\). The inside heat transfer coefficient, \(h_i\), is 15 Btu/hr-ft-\(^\circ\text{F}\), and the outside coefficient, \(h_o\), is 2.2 Btu/hr-ft-\(^\circ\text{F}\).

Draw (a) a schematic (1.0 pt) and, (b) the equivalent thermal circuit labeled with temperature and resistances. (1.0 pts) (c) Determine the heat transfer rate from the steam per foot of pipe length, if the surrounding air temperature is 65°F. (1.0 pts)

\[
\begin{align*}
R_i &= \frac{1}{h_i \cdot A} = \frac{1}{h_i \cdot (2\pi r_i L)} \\
R_o &= \frac{1}{h_o \cdot A} = \frac{1}{h_o \cdot (2\pi r_o L)} \\
R_0 &= \frac{1}{h_o \cdot (2\pi r_o L)} \\
R_1 &= \frac{1}{h_i \cdot A} = \frac{1}{h_i \cdot (2\pi r_i L)} \\
R_2 &= \frac{1}{2\pi k_{\text{steel}} L} \\
R_3 &= \frac{1}{2\pi k_{\text{mag}} L} \\
R_u &= \frac{1}{h_i \cdot (2\pi r_i L)} \\

C(c) \quad \phi &= \frac{T_{wi} - T_{wo}}{R \cdot ft} \\
&= \left(2\pi L \right) \cdot \frac{T_{wi} - T_{wo}}{\frac{1}{h_i \cdot r_i} + \frac{\ln \left(\frac{r_o}{r_i} \right)}{2\pi k_{\text{steel}} L} + \frac{\ln \left(\frac{r_o}{r_i} \right)}{2\pi k_{\text{mag}} L} + \frac{1}{h_o \cdot (2\pi r_o L)}} \\
\therefore \quad \frac{\phi}{L} &= \frac{2\pi (250 - 65)}{15 \text{ Btu/hr-ft-}^\circ\text{F}} \cdot \frac{(2.5 \text{ ft})}{(3.0 \text{ in})} + \frac{\ln \left(\frac{0.25}{0.2} \right)}{2\pi k_{\text{steel}} L} + \frac{\ln \left(\frac{3.25}{2.25} \right)}{2\pi k_{\text{mag}} L} + \frac{0.041}{(2.2) (2\pi r_o L)} \\
&= 105.8 \text{ Btu/hr-ft}
\end{align*}
\]

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3. If the maximum heat transfer rate through the composite wall is 900W/m², how thick must the layer 2 be? (2.0 pts)

\[ q = \frac{\Delta T_{max}}{R_{tot}} = \frac{(T_1 - T_2)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}} = \frac{1250 - 310}{\frac{0.2}{1.0} + \frac{x}{0.07}} \]

\[ \therefore x = 0.059 \text{ m.} \]
4. A long copper \((k=223 \text{ Btu/hr-ft}^\circ\text{F})\) wire, 0.25in in diameter, was held in an air stream with a temperature \(T_\infty=100^\circ\text{F}\). After 30s, the average temperature of the wire increased from 50\(^\circ\text{F}\) to 80\(^\circ\text{F}\). Make an estimate of the average convection heat transfer coefficient \(h\). (40 pts)

\[\dot{q} = 555 \text{ Btu/ft}^2, \quad C = 0.092 \text{ Btu/lb}^\circ\text{F}\]

\[h = \frac{T_\infty - T}{\dot{q} / A} = \frac{0.25 \text{ ft}}{(223 \text{ Btu/hr-ft}^\circ\text{F})} = 1.13 \times 10^{-4} \text{ Btu/hr-ft}^2^\circ\text{F}\]

\[Bi = \frac{hLc}{k} = \frac{0.25}{(223 \text{ Btu/hr-ft}^\circ\text{F})} = 2.8 \times 10^{-4} \text{ Btu/hr-ft}^2^\circ\text{F}\]

Let's get \(Bi = 0.1\) which is reasonable limiting value of \(Bi\) for a lumped capacitance method to be valid, and solving for \(h\)

\[h = \frac{C}{2.8 \times 10^{-4}} = 357 \text{ Btu/hr-ft}^2^\circ\text{F}\]

Thus, a lumped capacitance method will be sufficiently accurate so long as \(\dot{q} < 357 \text{ Btu/hr-ft}^2^\circ\text{F}\).

\[(5.5) \quad \frac{\rho V C}{A s} \ln \frac{\theta_i}{\theta} = t\]

\[h = \frac{\rho V C}{A s} \ln \frac{\theta_i}{\theta} \]

\[= \frac{(555 \text{ Btu/ft}^2) (0.092 \text{ Btu/lb}^\circ\text{F}) (\frac{\text{TL}^2}{C})}{(355) (\text{TL})} \ln \frac{80 - 10^\circ\text{F}}{50 - 10^\circ\text{F}} = 29.2 \text{ Btu/hr-ft}^2^\circ\text{F} < 357 \text{ Btu/hr-ft}^2^\circ\text{F}\]

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5. Determine the heat transfer rate per unit length from a 2.0-in outer diameter isothermal pipe located 5-in below the surface of a thick concrete slab. The slab is very wide and very thick, resulting in the two-dimensional problem. Thermal conductivity of concrete is 0.44 Btu/hr·ft·°F. (3.0 pts)

2-Dimensional conduction, shape factor problem.

Table 4.1 (p 209), Case 2.

\[ r = 5 \text{ in} \]
\[ R = 30/2 \cdot \text{ Second case of case 2} \]

\[ D = 2 \text{ in} \]

\[ S = \frac{2 \pi L}{\ln\left(\frac{4 \pi L}{D}\right)} = \frac{(2 \pi L)(5 \text{ in})}{\ln\left(\frac{4 \times 5 \text{ in}}{2 \text{ in}}\right)} = 2.727 \text{ in} \]

\[ q = S \cdot k (T_1 - T_2) \]
\[ = (2.727)(2\text{ in}) \times (0.44 \text{ Btu/ hr· ft· °F}) \times (110 - 70) \text{ °F} \]

\[ \frac{q}{L} = \frac{(2.727)(0.44)(110 - 70)}{2 \text{ in}} = 48.62 \text{ Btu/ hr· ft} \]

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6. Two long pieces of 1.0-in diameter copper rod \((k=207 \text{ Btu/hr-ft-}^\circ\text{F})\) are to be silver soldered together end to end. The surrounding air temperature is \(80^\circ\text{F}\), and the melting point of the solder is \(1200^\circ\text{F}\). If the heat transfer coefficient between the copper and the air is \(3.0 \text{ Btu/hr-ft}^2\cdot^\circ\text{F}\), find the minimum energy input to keep the soldered surface at \(1200^\circ\text{F}\). (5.0 pts)

Choosing the junction of the two rods as the origin of \(x\), we have an extended fin of constant cross section. By symmetry, the total heat transfer \(=\) energy input \(=\) twice the heat transfer into \(x>0\). Thus,

Table B.4. Case D. \(\rho = \pi \rho = \pi \text{-in.} \cdot \frac{\pi}{12} \text{ ft}\)

\[
q = \sqrt{h \cdot P \cdot \frac{\pi}{12}} \cdot \Theta_0 = \sqrt{(3.0 \text{ Btu/hr-ft}^2\cdot^\circ\text{F}) \cdot \left(\frac{\pi}{12}\right) \cdot (207 \text{ Btu/hr-ft} \cdot ^\circ\text{F}) \cdot \left(\frac{\pi}{12} \cdot \frac{1}{\text{in.}^2}\right) \cdot (1200 - 80 \circ \text{F})^\circ \text{F}}
\]

\(= 1502.8 \text{ Btu/hr}\)

\(\therefore\) required heat \(= 2q\)

\(= 3005.6 \text{ Btu/hr}\)

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7. Determine the percentage increase in heat transfer associated with attaching aluminum fins of rectangular profile to a plane wall. The fins are 10 cm long, 1.0 mm thick, and are equally spaced at a distance of 5 mm. The convection coefficient associated with the bare wall is 40 W/m²·K, while that resulting from attachment of the fins is 30 W/m²·K.

**Evaluating Fin Parameters**

\[ L_c = L + \frac{d}{2} = 1.0 + 0.001 = 1.001 \text{ m} \]

\[ A_f = L_c 	imes t = (1.001 \text{ m}) 	imes (0.001 \text{ m}) = 1.005 \times 10^{-4} \text{ m}^2 \]

\[ (L_c - \frac{d}{2})K_{eq} = \left(1.005 \text{ m} \right) \left( \frac{30 \text{ W}}{m^2 \cdot K} \right) \left( \frac{1}{240 \text{ W/m} \cdot \text{K}} \right) \left(1.005 \times 10^{-4} \text{ m}^2 \right) \]

\[ = 1.129 \]

Fig. 3.18 \[ \eta_f \approx 0.52 \text{ Hence} \]

\[ \eta_f = \eta_f \cdot \eta_{\text{max}} = (0.52) \left( \frac{30 \text{ W}}{m^2 \cdot K} \cdot \frac{(2)(0.1)(w) \cdot \theta_b}{3.12 \text{ W/m} \cdot \text{K}} \right) \]

\[ \eta_f = 1.29 \]

For multiple fins,

\[ \eta_f = \frac{N_{AF} + A_b}{A_f} \]

\[ \eta_f = \frac{N_{AF} + A_b}{A_f} \]

\[ At = NAF + A_b \]

\[ At = 1 \text{ m}^2 \]

\[ q_{fs} = \eta_f \cdot (hA_f \cdot \theta_b) \]

\[ = \left[ 1 - \frac{NAF}{A_f} \right] (hA_f \cdot \theta_b) \]

\[ = hA_f \cdot \theta_b - NAF \cdot hA_f \cdot \theta_b + \frac{hA_f \cdot \theta_b}{N_{AF}} = N_{AF} \cdot \theta_b \]

\[ \text{in 1m height wall} \rightarrow N = 166 \]

\[ q_f = \eta_f \cdot \left( (3.12 \text{ W/m} \cdot \text{K}) \cdot (0.001 \text{ m} \cdot \text{K}) \right) \]

\[ q_f = \eta_f \cdot \left( (3.12 \text{ W/m} \cdot \text{K}) \cdot (0.001 \text{ m} \cdot \text{K}) \right) \]

\[ = 542.94 \text{ W/K} \]

without fins, \[ g_{wall} = \frac{h_{wall} \cdot (1 \text{ m} \cdot \text{W}) \cdot \theta_b}{40 \text{ W/K}} = \frac{(40 \text{ W/m} \cdot \text{K}) \cdot \theta_b}{40 \text{ W/K}} \]

\[ = 1 \text{ W/K} \]

\[ \text{percentage increase in heat transfer is} \]

\[ \frac{q_f - g_{wall} \text{ only}}{g_{wall} \text{ only}} = \frac{(542.94 - 40) \text{ W/K}}{40 \text{ W/K}} = 12.57 = 1257\% \]

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