MAE 3314: Heat Transfer
Homework #6 (Due date – 11/1/07) Key Assignment

1. [10 pts] Forced air at $T_a=25^\circ C$ and $V=10\text{m/s}$ is used to cool electronic elements on a circuit board. One such element is a chip, 4mm by 4mm, located 120mm from the leading edge of the board. A chip is dissipating 30 mW. Experiments have revealed that flow over the board is distributed by the elements and that convection heat transfer is correlated by the elements and that convection heat transfer is correlated by an expression of the form $Nu_x = 0.04 \text{Re}_x^{0.85} \text{Pr}^{1/3}$. The cooling condition listed above was designed at atmospheric pressure (= 1 atm).

(a) Estimate the surface temperature of the chip located 120 mm from the leading edge of the board when the board is operated at 76.5 kPa.

(b) It is desirable for the chip operating temperature to be independent of the location. What air velocity is required for operation at 76.5 kPa if the chip temperature is to be the same as at 1 atm?

* At 1 atm, $35^\circ C$ ($= T_f = (45^\circ C+25^\circ C)/2$), air has values of: $k = 0.0269 \text{W/m$\cdot$K}$, $\nu = 16.69 \times 10^{-6} \text{ m}^2/\text{s}$, $Pr = 0.706$ (From Table A.4)

* Assume that the average heat transfer coefficient for the chip surface is equivalent to the local value at $x = L$.

* Assume that air has ideal gas behavior.

2. [10 pts] Nitrogen at $100^\circ F$ and 1 atm flows at a velocity of 10fps. A flat plate 6 in wide, at a temperature of $200^\circ F$, is aligned parallel to the direction of flow. At a position 4 ft from the leading edge, determine the following:

(a) $\delta$
(b) $\delta_t$
(c) $C_{fx}$
(d) $\bar{C}_{f,t}$
(e) $h_x$
(f) $\bar{h}_L$
(g) total drag force
(h) total heat transfer

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Known:
- \( \text{Nu_x expression, interms of Re & Pr} \)
- Flow condition
- \( \dot{E}_g = 30 \text{mW} \)

Find:
- Surface temperature of chip surface (\( T_s \)) under atmospheric pressure of 76.5 kPa

Assumption:
1. St. St. condition.
2. Chip Surf. is isothermal
3. The average heat transfer coefficient (\( \overline{h} \)) for the chip surface is equivalent to the local value at \( x=L \) (\( \overline{h}\times L \))
4. Ideal gas behavior

Properties:
- Table A-4. \( \text{air} \) \( \rho = 1 \text{atm} \), \( T_f = (45^\circ\text{C} + 25^\circ\text{C})/2 = 35^\circ\text{C} \)
- \( \dot{E}_g = 0.0269 \text{ W/m-K} \)
- \( V = 16.69 \times 10^{-6} \text{ m/s} \)
- \( Pr = 0.706 \)

Solution:
(a) From an energy balance on the chip
\[
\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_{\text{gen}} = \dot{E}_{\text{st}}
\]
\[
\dot{q}_{\text{conv}}\quad 30 \text{mW}
\]
\[
\therefore \dot{q}_{\text{conv}} = \dot{E}_g = 30 \text{mW}.
\]
\[
T_s = T_w + \frac{\dot{q}_{\text{conv}}}{\overline{h} \cdot \text{A}_{\text{chip}}}
\]
\[
\text{A}_{\text{chip}} = l^2 \quad l = 4 \text{mm}
\]

From the Assumption \( \overline{h} \approx \overline{h}_\times L \)
\[
\text{Nu}_x = \frac{\overline{h}_x \cdot x}{\kappa} = 0.04 \text{ Re}^{0.85} \text{ Pr}^{1/3}
\]

To calculate \( \text{Re}_x \), we need \( V \) (dynamic viscosity)
\[
V = \frac{k}{\rho}
\]

We assumed that air has ideal gas behavior,
\[
\rho = \frac{P}{RT} \quad \therefore \frac{V_1}{V_2} = \frac{P_2}{P_1} = \frac{P_2}{P_1} \quad \therefore \frac{V_1}{\text{atm}} = \frac{V_2}{76.5 \text{kPa}} \cdot \frac{1 \text{ atm}}{76.5 \text{kPa}}
\]
\[ \gamma = (16.69 \times 10^{-6} \text{ m}^2/\text{s}) \cdot \left( \frac{1 \text{ atm} = 101.3 \text{ kPa}}{76.5 \text{ kPa}} \right) = 22.10 \times 10^{-6} \text{ m}^2/\text{s} \]

\[ \text{Pr} = \frac{\nu}{\alpha} = \left( \frac{\mu}{\frac{k}{\rho C_p}} \right) = \frac{\nu C_p}{k} \Rightarrow \text{independent of pressure change} \]

So at 76.5 kPa

\[ \gamma = 22.10 \times 10^{-6} \text{ m}^2/\text{s} \]

\[ k = 0.0269 \text{ W/m.K} \]

\[ \text{Pr} = 0.706 \]

\[ \text{Re}_x = \frac{U_0 \cdot L}{\nu} = \frac{(10 \text{ m/s}) \cdot (120 \text{ mm})}{(22.10 \times 10^{-6} \text{ m}^2/\text{s})} = 5.43 \times 10^4 \]

\[ \text{Nu}_x = \frac{h_x \cdot L}{k} = 0.04 \text{ Re}_x^{0.85} \cdot \text{Pr}^{1/3} \]

\[ = (0.04)(5.43 \times 10^4)^{0.85} \cdot (0.706)^{1/3} = 376.73 \]

\[ h_x = (\text{Nu}_x) \cdot \left( \frac{k}{L} \right) \]

\[ = (376.73) \cdot \left( \frac{0.0269 \text{ W/m.K}}{120 \text{ mm}} \right) = 84.45 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \]

\[ T_s = T_w + \frac{q_{\text{conv}}}{h_x \cdot A_{\text{chf}}} \]

\[ = 25^\circ C + \frac{30 \text{ mW}}{(84.45 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}) \cdot (4 \text{ mm})^2} = 47.2^\circ C \]

(b) For 1 atm operation,

\[ h_x = (0.04) \cdot \left( \frac{0.0269 \text{ W/m.K}}{120 \text{ mm}} \right) \cdot \left( \frac{10 \text{ m/s}}{16.67 \times 10^{-6} \text{ m}^2/\text{s}} \right)^{0.85} \cdot (0.706)^{1/3} \]

\[ = 107 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \]

\[ \text{in order to achieve this h value at 76.5 kPa}, \]

\[ h_x = 107 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} = 0.04 \left( \frac{0.0269 \text{ W/m.K}}{120 \text{ mm}} \right) \cdot \left( \frac{V \cdot 120 \text{ mm}}{22.10 \times 10^{-6} \text{ m}^2/\text{s}} \right)^{0.85} \cdot (0.706)^{1/3} \]

\[ \Rightarrow V = 13.2 \text{ m/s}. \]
Question 2

$N_2 \rightarrow \frac{\delta}{S_t}$

$\nu = 100\,\text{fps}$
$P = 1\,\text{atm}$
$T_w = 100\,\text{°F}$
$x = 4\,\text{ft}$

$T_s = 200\,\text{°F}$

Properties:

$N_2 @ P = 1\,\text{atm}$

$T_p = \frac{(100 + 200)}{2} = 150\,\text{°F} = 338.56\,\text{K}$

Table A-4 @ 350K

- $\rho = 0.9625\,\text{kg/m}^3$
- $c = 1.042\,\text{kJ/kg.K}$
- $\mu = 200.0 \times 10^{-7}\,\text{N.s/m}^2$
- $\nu = 20.78 \times 10^{-6}\,\text{m}^2$/s
- $k = 29.3 \times 10^{-3}\,\text{W/m.K}$
- $\alpha = 29.2 \times 10^{-6}\,\text{m}^2$/s
- $Pr = 0.711$

Table A-4 @ 338.56K

- $\rho = 0.9993\,\text{kg/m}^3$
- $\nu = 19.66 \times 10^{-6}\,\text{m}^2$/s
- $k = 28.52 \times 10^{-3}\,\text{W/m.K}$
- $Pr = 0.712$

$\downarrow$ Unit conversion,

- $\rho = 0.0620\,\text{lbm/ft}^3$
- $\nu = 2.11506 \times 10^{-5}\,\text{ft}^2$/s
- $k = 16.478\,\text{Btu/ft.h.°F}$

at $x = 4\,\text{ft}$

$Re_x = \frac{\rho \cdot \nu \cdot L}{\nu} = \frac{(10\,\text{ft/s}) \cdot (4\,\text{ft})}{211.506 \times 10^{-6}\,\text{ft}^2/s} = 1.8912 \times 10^5 < Re_{x,c} = 5 \times 10^5$

"Flow is fully laminar till $x = 4\,\text{ft}"

Remember the flow condition is "Laminar Flow over Isothermal Plate" Textbook 7.2.1

(a) Equation (7.19)

$\delta = \frac{5.0}{\sqrt{\nu \cdot \nu}} = \frac{5.0}{\sqrt{Re_x}} = \frac{(5) \cdot (4\,\text{ft})}{\sqrt{1.8912 \times 10^5}} = 0.0460\,\text{ft}$

(b) Equation (7.24)

$\delta = \frac{5}{Pr^\frac{1}{2}} \Rightarrow \delta = \frac{5}{Pr^\frac{1}{2}} = \frac{0.0460\,\text{ft}}{0.712^\frac{1}{2}} = 0.05152\,\text{ft}$

(c) Equation (7.20)

$C_{f,x} = \frac{\tau_{w,x}}{\frac{1}{2} \rho \nu \cdot \nu} = 0.664 \left( Re_x \right)^{-\frac{1}{2}} = 0.664 \left( 1.8912 \times 10^5 \right)^{-\frac{1}{2}}$

$C_{f,x} = 0.001526 = 1.526 \times 10^{-3}$
(d) Equation (7.29)
\[ \bar{C}_{f,x} = 1.328 \text{Re}_x^{-1/2} = 2 \cdot C_{f,x} \]
\[ = 2 \times 0.001526 \]
\[ = 0.003053 = 3.053 \times 10^{-3} \]

(e) Equation (7.23)
\[ N_{ux} = \frac{h_{x,L}}{k} = 0.332 \text{Re}_x^{1/2} \text{Pr}^{1/3} \quad \text{for } \text{Pr} = 0.712 \geq 0.6. \]
\[ i.e., \quad h_{x,L} = \frac{N_{ux} \cdot k}{x} = \left( \frac{\text{Re}_x^{1/2} \text{Pr}^{1/3}}{x^{10^{-3}}} \right) \left( 0.332 \right) \left( 1.8912 \times 10^{-5} \right)^{1/2} \left( 0.712 \right)^{1/3} \]
\[ = 0.5313 \text{ Btu/hr ft}^2 \circ F \]

(f) Equation (7.30)
\[ \bar{N}_{ux} = \frac{\bar{h}_{x,L}}{k} = 0.664 \text{Re}_x^{1/2} \text{Pr}^{1/3} \]
\[ = 2 \cdot \bar{N}_{ux} \]
\[ \therefore \bar{h}_{x,L} = 2 \cdot h_{x,L} = 1.0626 \text{ Btu/hr ft}^2 \circ F \]

(g) Total drag force \( F_0 = \bar{C}_{t,s} \cdot A_s \)

Equation (7.28)
\[ \bar{C}_{f,s} = \frac{\bar{C}_{t,s} \cdot \bar{u}^2}{2} \rightarrow F_0 = \bar{C}_{f,s} \cdot \bar{u}^2 \]
\[ = \left( \bar{C}_{f,s} \cdot \frac{\bar{u}^2}{2} \right) (A_s) \]
\[ = (3.053 \times 10^{-3}) \cdot \left( \frac{0.0620 \text{ lbm}}{\text{ft}^3} \cdot (10 \text{ ft/s})^2 \right) \cdot \left( \frac{4 \text{ ft}}{0.5 \text{ ft}} \right) \]
\[ = 18.929 \times 10^{-3} \text{ lbm ft/s}^2 \]
Total heat transfer

\[ q = \bar{h} \cdot A \cdot (T_s - T_w) \]

\[ = (1.0626 \text{ Btu/hr ft}^2 \text{ of}) \cdot (4\text{ ft} \times 6\text{ in}) \cdot (200^\circ F - 100^\circ F) \]

\[ = 212.52 \text{ Btu/hr} \]