MAE 2314  FLUID MECHANICS

SUMMER 2009

DEPARTMENT OF MECHANICAL
AND
AEROSPACE ENGINEERING

UNIVERSITY OF TEXAS AT ARLINGTON

EXAM #2

CLOSED BOOK AND NO NOTES
Only Nonprogrammable Calculators are allowed

AUGUST 5, 2009
Time Limit : 1 hr 45 min

This exam has 7 pages.

LAST NAME: [Signature]
FIRST NAME: 

Announcement on types: 
\[ t = \frac{10^3}{10^6} \text{ yr} \]
Unless otherwise stated.
1. (20pts) A vertical jet of water leaves a nozzle at a speed of 10 m/s and a diameter of 20 mm. It suspends a plate having a mass of 1.5 kg as indicated. What is the vertical distance h? \( (\rho = 999 \text{ kg/m}^3; \ g = 9.81 \text{ m/s}^2) \)

\[
W = m\ V_1 \Rightarrow V_1 = \frac{W}{m} = \frac{mg}{m} = g \cdot \frac{(15 \text{ kg})(9.8 \text{ m/s})}{3.14 \text{ kg/s}}
\]

\[
= 4.68 \text{ m/s}
\]

Applying Bernoulli’s Eq. between \( o \) to \( y \):

\[
\frac{P_0}{\rho} + \frac{V_3^2}{2} + g \cdot z_3 = \frac{P_1}{\rho} + \frac{V_2^2}{2} + g \cdot z_1
\]

\[
h = (z_1 - z_3) = \frac{V_2^2 - V_1^2}{2 \ g}
\]

\[
= \frac{(10^2 - 4.68^2) \text{ m}^2/\text{s}^2}{2 \ (9.8) \text{ m/s}^2} = 3.88 \text{ m}
\]
2a. (5pts) The velocity components in a steady, incompressible, two-dimensional flow field are: \( u = 2x; \ v = -2y \)
For this flow field find the equation of the streamline through the point (1,1).

\[
\begin{align*}
\frac{dy}{dx} = \frac{v}{u} = \frac{-2y}{2x} &= -\frac{y}{x} \\
\Rightarrow \quad \frac{dy}{y} = \frac{-dx}{x} &\quad \Rightarrow \quad \ln y = -\ln x + \ln c \\
\Rightarrow \quad y &= cx \quad \text{The streamline passes through (1,1)}
\end{align*}
\]

\( c = 1 \Rightarrow xy = 1 \)

2b. (5pts) An incompressible velocity field is given by \( u = a(x^2-y^2) \), \( v \) unknown, \( w = b \), where \( a \) and \( b \) are constants. What must the form of the velocity component \( v \) be?

\[
\begin{align*}
u &= \frac{\partial}{\partial y} \left( a(x^2-y^2) \right) \\
\text{Incompressible flow} &\Rightarrow \triangledown \cdot \vec{v} = 0 \\
\Rightarrow \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \\
\Rightarrow \quad 2ax + \frac{\partial v}{\partial y} + 0 &= 0 \\
\Rightarrow \quad \frac{\partial v}{\partial y} &= -2ax \\
\Rightarrow \quad v &= -2axy + f(x,z,t)
\end{align*}
\]

2c. (5pts) The stream function for a two-dimensional, incompressible flow field is given by \( \psi = x^2 + y^2 \). Is this an irrotational flow field? Explain.

\[
\begin{align*}
\frac{\partial \psi}{\partial y} &= 2x \\
\frac{\partial \psi}{\partial x} &= 2y \\
\Rightarrow \quad \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} &= 2x - 2y = 2(\psi) - 2(-\psi) = 4 \Rightarrow \text{Irrotational}
\end{align*}
\]
3. (15pts) The transient temperature distribution in a fluid is given by \( T = (10x + 5y)(1 + t) \), where \( x \) and \( y \) are the horizontal and vertical coordinates in meters, \( T \) in degrees centigrade and \( t \) is time in seconds. Determine the time rate of change of temperature of a fluid particle located at \((1,2)\) at \( t = 5 \):

(i) travelling horizontally in the \( x \)-direction (i.e. \( \theta = 0^\circ \)) at \( 1 \) m/s.

\[
\frac{dT}{Dt} = \frac{dT}{dt} + u \frac{dT}{dx} = 80 \degree C/s
\]

\[
\left(10x + 5y\right) \quad \frac{1}{10(1+t)}
\]

\[
= 80
\]

(ii) travelling diagonally (i.e. \( \theta = 45^\circ \)) at \( 1 \) m/s.

\[
\frac{dT}{Dt} = \frac{dT}{dt} + \left( \frac{\sqrt{2}}{2} u \right) \frac{dT}{dx} + \left( \frac{\sqrt{2}}{2} v \right) \frac{dT}{dy}
\]

\[
\frac{\sqrt{2}}{2} u = \frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2} v = \frac{\sqrt{2}}{2}
\]

\[
= 80 + 30 \sqrt{2} + 15 \sqrt{2}
\]

\[
= 80 + 45 \sqrt{2}
\]

\[
83.64 \degree C/s
\]

(iii) staying stationary.

\[
\frac{dT}{Dt} = \frac{dT}{dt} = 20 \degree C/s
\]
4a (10 pts). Determine the acceleration \((a_x, a_y, a_z)\) of a particle at \((1, 2, 3)\) at \(t=4\) in the velocity field \(\mathbf{V} = 3\mathbf{i} + x\mathbf{j} + y^2\mathbf{k}\).

\[ a_x = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = 3 \]

\[ a_y = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = 3t^2 + 3ty^2 

\]

\[ a_z = \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = y^2 + 3txyz 

\]
4b. (15pts) An incompressible viscous fluid is placed between two large parallel plates. The bottom plate is fixed and the upper plate moves with a constant velocity, \( U \). For these conditions the velocity distribution between the plates is linear, and can be expressed as \( u = U y/b \). Determine: (a) the volumetric dilatation rate, (b) the vorticity, and (c) the rate of angular deformation.

(a) Volumetric dilatation rate:

\[
\nabla \cdot \mathbf{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \Rightarrow \quad \text{Incompressible}
\]

(b) Vorticity:

\[
\omega_y = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{U}{b}
\]

(c) Rate of angular deformation:

\[
\dot{\theta} = \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = \frac{U}{b}
\]
\[ \rho = \frac{10^3 \text{ kg/m}^3}{\text{m}^3} \]

5. (25 pts) A horizontal flow with an inlet diameter of 1 m is divided and has equal flow rates from each outlet. The outlets have diameter of 0.5 m. The inlet gage pressure is 200 kPa and the inlet flow rate is 5 m³/s. Determine the reaction force on the divided flow that must be absorbed by a support system. **Define your control volume and label your nodes.**

\[
\begin{align*}
V_1 &= \frac{V_1}{A_1} = \frac{5 \text{ m}^3/\text{s}}{\frac{\pi}{4} \text{ m}^2} = 6.37 \text{ m/s} \\
V_2 &= \frac{V_2}{A_2} = \frac{2.5 \text{ m}^3/\text{s}}{\frac{\pi}{4} \cdot 0.5^2 \text{ m}^2} = 12.73 \text{ m/s} \\
V_3 &= \vec{V}_2
\end{align*}
\]

**Bernoulli's Principle:**

\[
\frac{p_1}{\rho} + \frac{1}{2} \rho V_1^2 + \rho g z_1 = \frac{p_2}{\rho} + \frac{1}{2} \rho V_2^2 + \rho g z_2 = \frac{p_3}{\rho} + \frac{1}{2} \rho V_3^2
\]

(assume change \( z \approx 0 \))

\[
\Rightarrow p_2 = p_3 = p_1 + \frac{1}{2} \rho (V_1^2 - V_2^2)
\]

\[
\approx 1393 \text{ kPa}
\]

**Force balance:**

\[
\sum F_x = P_1 A_1 - P_3 A_3 \cos 45° + F_x
\]

\[
\sum F_y = F_y + P_3 A_3 \cos 45° - P_2 A_2 = m_2 V_2 - m_3 V_3 \cos 45°
\]

\[
\sum F_z = 0
\]

\[
F_x = 17.33 \text{ kN}
\]

\[ F_y = 17.33 \text{ kN} \]