An infinite plate is moved over a second plate on a layer of liquid. For small gap width, \(d\), we assume a linear velocity distribution in the liquid. The liquid viscosity \(\mu\) is \(1.36 \times 10^{-5}\) lbf-s/ft\(^2\) and its specific gravity is 0.88. Determine (a) Kinematic viscosity \(v\) in SI units. (b) Shear stress on the upper plate in lbf/ft\(^2\). (c) Shear stress on the lower plate in Pa. (d) The direction of each shear stress calculated in parts (b) and (c).

Given: \(\mu = 1.36 \times 10^{-5}\) lbf-s/ft\(^2\).

(a) Kinematic Viscosity, \(v = \frac{\mu}{\rho} = \frac{\mu}{\frac{\text{SG} \times \rho_{H_2O}}{\text{lbf} \cdot \text{s} / \text{ft}^2}}\)

\[
= 1.36 \times 10^{-5} \left(\frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right) \times \frac{\text{ft}^3}{0.88 \times 1.94 \text{ slug} \cdot \text{ft}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \times \frac{(0.3) \text{ m}^2}{\text{ft}^2}
\]

\[
= 7.4 \times 10^{-7} \text{ m}^2/\text{s}
\]

(Ans)

(b) \(T_{\text{upper}} = \mu \frac{d}{dy} \left|_{y=d}\right. \Rightarrow \mu \left(\vec{n} \cdot \nabla V\right)_{y=d} \Rightarrow \text{in vector form:}

\[
= \mu \left(0, -1\right) \cdot \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}\right)
\]

\[
= -\mu \frac{\partial V}{\partial y} = -1.36 \times 10^{-5} \left(\frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right) \times \frac{0.3 \times 1000}{0.3}
\]

\[
= -0.0136 \text{ lbf/ft}^2
\]

(Ans)

(c) \(T_{\text{lower}} = \mu \left(0, 1\right) \cdot \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}\right)

\[
= \mu \frac{\partial V}{\partial y} = 0.0136 \left(\frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right) \times 4.45 \text{ N} \times \frac{\text{ft}^2}{\text{lbf} \cdot \text{m}^2}
\]

\[
= 0.651 \frac{\text{N}}{\text{m}^2} = 0.651 \text{ Pa}
\]

(Ans)

(d) Direction: b) \(\Rightarrow \text{in negative X-direction.}

\(\Rightarrow \text{in positive X-direction.}\)
A 50-cm \times 30\text{-}cm \times 20\text{-}cm block weighing 150 N is to be moved at a constant velocity of 0.8 m/s on an inclined surface with a friction coefficient of 0.27. (a) Determine the force F that needs to be applied in horizontal direction. (b) If a 0.4-mm-thick oil film with a dynamic viscosity of 0.012 Pa\text{-}s is applied between the block and the inclined surface, determine the percent reduction in the required force.

\[ A = 50 \times 20 \text{ cm}^2 = 1 \text{ m}^2 \]

\[ \text{Normal force, } N = F \sin 20^\circ + W \cos 20^\circ \]

\[ \text{Force of friction } = \mu N = \mu (F \sin 20^\circ + W \cos 20^\circ) \]

For equilibrium in x-direction:

\[ F \cos 20^\circ = W \sin 20^\circ + \mu (F \sin 20^\circ + W \cos 20^\circ) \]

\[ F \cos 20^\circ - \mu F \sin 20^\circ = W \sin 20^\circ + \mu W \cos 20^\circ \]

\[ F(0.9396 - 0.09234) = 51.303 + 28.06 \]

\[ F = \frac{89.36}{0.8472} = 105.5 \text{ N} \]

Ans.

\[ \mu_{01} = 0.012 \text{ Pa\text{-}s} \]

\[ h = 0.4 \text{ mm} = 0.0004 \text{ m} \]

\[ \tau_w = \mu_{01} \left. \frac{d\mu}{dy} \right|_{y=h} = 0.012 \times 0.8 \times \frac{0.0004}{0.0004} = 0.24 \text{ N/m}^2 \]

\[ F_x = \tau_w A = 0.24 \times 0.1 = 0.024 \text{ N} \]

For equilibrium in x-direction:

\[ F \cos 20^\circ = F_x + 150 \sin 20^\circ \]

\[ F = \frac{0.024 + 150 \times 0.3420}{\cos 20^\circ} = 57.15 \text{ N} \]

Ans.

\[ \% \text{ Reduction} = \frac{105.5 - 57.15}{105.5} \times 100 = 45.8 \% \]
Consider the flow of a fluid with viscosity $\mu$ through a circular pipe. The velocity profile in the pipe is given as $u(r) = u_{\text{max}}(1 - r^2/R^2)$, where $u_{\text{max}}$ is the maximum flow velocity, which occurs at the centerline; $r$ is the radial distance from the centerline; and $u(r)$ is the flow velocity at the position $r$. Develop a relation for the drag force exerted on the pipe wall by the fluid in the flow direction per unit length of the pipe.

$$u(r) = u_{\text{max}}[1 - (\frac{r}{R})^2]$$

$$\tau_w = \mu [\mathbf{\hat{n}} \cdot \nabla u] = [\langle -1, 0 \rangle \cdot \frac{\partial u(r)}{\partial \theta}, \frac{\partial u(r)}{\partial \phi}]$$

$$= -\mu \left. \frac{\partial u(r)}{\partial \theta} \right|_{\theta = R}$$

$$= -\mu \cdot u_{\text{max}} \left[ 0 - \frac{\eta R^{n-1}}{R^n} \right] = \mu u_{\text{max}} \frac{\eta R^{n-1}}{R^n} \bigg|_{R = R}$$

$$\tau_w = \mu u_{\text{max}} \frac{\eta R^{n-1}}{R^n} = \eta \frac{\mu u_{\text{max}}}{R}$$

$\tau_w$ is the shear force acting on unit area on the inner wall.

So Total Drag force acting

$$D = \tau_w \times \text{Area}$$

$$= \eta \frac{\mu u_{\text{max}}}{R} \cdot 2\pi R L$$

$$D = \frac{\eta \mu u_{\text{max}} \cdot 2\pi R}{R} = 2\pi \eta \mu u_{\text{max}}$$
The viscosity of a fluid is to be measured by a viscometer constructed of two 3-ft-long concentric cylinders. The inner diameter of the outer cylinder is 6 in, and the gap between the two cylinders is 0.05 in. The outer cylinder is rotated at 250 rpm, and the torque is measured to be 1.2 lbf-ft. Determine the viscosity of the fluid.

\[ L = 30 \text{ ft} \]
\[ R = 6/2 = 3 \text{ in} = 0.25 \text{ ft} \]
\[ \text{Gap, } d = 0.05 \text{ in} = 0.05/12 \text{ ft} \]

Angular velocity of outer cylinder, \( \omega = \frac{2\pi \times 250}{60} = \frac{25\pi}{3} \text{ rad/s} \)

Tangential velocity on inner surface of the outer cylinder, \( v_0 = \omega r = \frac{25\pi}{3} \times 0.25 = 6.25\pi \text{ ft/s} \)

\( \frac{dv}{dr} = \frac{6.25\pi}{0.05/12} = \frac{6.25\pi \times 12^2}{2 \times 0.05} = 500\pi \text{ s}^{-1} \)

Total force = \( T \cdot \text{Area} = \mu \frac{dv}{dr} \cdot \text{Area} \)

\[ = \mu \times 500\pi \times 2\pi R L = \mu \times 500\pi \times 2\pi \times 0.25 \times 3 \]
\[ = 750\pi \mu \text{ lbf} \]

Torque, \( T = \text{force} \times R = 750\pi^2 \mu R \).

But we have been given \( T = 1.2 \text{ lbf-ft} \).

\[ 750\pi^2 \mu R = 1.2 \]

\[ \mu = \frac{1.2}{750\pi^2 R} = \frac{1.2 \times 4}{750\pi^2} = 0.000648 \text{ lb-s/ft}^2 \]
#5.
In regions far from the entrance, fluid flow through a circular pipe is one-dimensional, and the velocity profile for the laminar flow is given by \( u(r) = \frac{u_{\text{max}}}{r^2} \), where \( R \) is the radius of the pipe, \( r \) is the radial distance from the center of the pipe, and \( u_{\text{max}} \) is the maximum flow velocity, which occurs at the center. Obtain (a) a relation for the drag force applied by the fluid on a section of the pipe of length \( L \) and (b) The value of the drag force for water flow with \( R = 0.08 \) m, \( L = 15 \) m, \( u_{\text{max}} = 5 \) m/s and \( \mu = 0.0010 \) kg/m·s.

\[
\begin{align*}
\tau_w &= \mu \left[ <1,0,0>, \frac{\partial u}{\partial r}, \frac{\partial^2 u}{\partial r^2} \right]_{r=R} \\
&= -\mu \frac{\partial u}{\partial r} \bigg|_{r=R} \\
&= -\mu u_{\text{max}} \left[ 0 - \frac{200}{R^2} \right] = 2\mu u_{\text{max}} \frac{L}{R^2} \bigg|_{r=R} \\
\tau_w &= 8\mu u_{\text{max}} \frac{L}{R^2} = \frac{2\mu u_{\text{max}}}{R} \\
\text{Drag force} &= \tau_w \times \text{area} = 8\mu u_{\text{max}} \frac{2\pi RL}{R} \\
\text{(a)} \quad D &= 4\mu \pi u_{\text{max}} L \\
\text{Ans} \\
\text{(b)} \quad D &= 4 \times 0.001 \times \pi \times 5 \times 15 \\
&= 0.3 \times \pi \text{ N} \\
&= 0.942 \text{ N} \quad \text{Ans}
\end{align*}
\]
#6.

The velocity distribution for the laminar flow between parallel plates is given by

$$\frac{u}{u_{\text{max}}} = 1 - \left(\frac{2y}{h}\right)^2$$

Where \( h \) is the distance separating the plates and the origin is placed midway between the plates. Consider the flow of water with \( u_{\text{max}} = 0.10 \text{ m/s} \) and \( h = 0.25 \text{ mm} \). Calculate the shear stress on the upper plate and give its direction. \((\mu \text{ of water is } 1.14 \times 10^{-3} \text{ N·s/m}^2)\)

$$u = u_{\text{max}} \left[1 - \left(\frac{2y}{h}\right)^2\right]$$

$$h = 0.25 \text{ mm} = 0.00025 \text{ m}$$

$$\mu_{\text{water}} = 1.14 \times 10^{-3} \text{ N·s/m}^2$$

$$\tau_w = \mu \left[ \nabla \cdot u \right]_{\text{wall}} = \mu \left[ \langle 0, -1 \rangle \cdot \left\langle \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \right\rangle \right] = -\mu \frac{\partial u}{\partial y} \bigg|_{y = h/2}$$

$$= -\mu \times u_{\text{max}} \left[ 0 - \frac{8y}{h^2} \right] = \frac{8\mu u_{\text{max}}}{h^2} \bigg|_{y = h/2}$$

$$\tau_w = \frac{8\mu u_{\text{max}}}{h^2} \left(\frac{h}{2}\right) = \frac{4\mu u_{\text{max}}}{h}$$

$$\tau_w = 4 \times 1.14 \times 10^{-3} \times 0.1 \frac{\text{N}}{0.00025 \text{ m}} = 4.56 \times 10^{-3} \times 0.1 \times 10^5 \frac{\text{N}}{25} = \frac{4.56}{25} = 0.28 \text{ N/m}^2$$

Direction →
The velocity distribution for the laminar flow between parallel plates is given by
\[ \frac{u}{u_{\text{max}}} = 1 - \left( \frac{2y}{h} \right)^2 \]

Where \( h \) is the distance separating the plates and the origin is placed midway between the plates. Consider the flow of water with the maximum speed of 0.05 m/s and \( h = 1 \text{ mm} \). Calculate the force on a 1 m² section of the lower plate and give its direction.

Given \( u = u_{\text{max}} \left[ 1 - \left( \frac{2y}{h} \right)^2 \right] \)
\[ \tau = \mu \left[ \nabla \cdot (\nabla u) \right] \]
\[ \tau = \mu \left[ <0, 1> \cdot \left( \frac{2y}{h} \right) \right] \]
\[ \tau = \mu \frac{2y}{h} \]
\[ \tau_{\text{lower}} = \mu \frac{2y}{h} \bigg|_{y = -\frac{h}{2}} = \mu u_{\text{max}} \left[ 0 - \frac{2y}{h^2} \right]_{y = -\frac{h}{2}} = -\frac{8 \mu u_{\text{max}}}{h^2} \cdot \frac{-h}{2} = 4 \frac{\mu u_{\text{max}}}{h} \]

\[ F_{\text{lower}} = \tau_{\text{lower}} \times \text{Area} = \tau_{\text{lower}} \times 1 \]
\[ = 4 \frac{\mu u_{\text{max}}}{h} = 4 \times 1.14 \times 10^{-3} \times 0.05 = 0.0228 \text{ N} \]

Direction: \( \rightarrow \), positive \( x \)-dir.
Crude oil, with specific gravity $SG = 0.85$ and the viscosity $\mu = 2.15 \times 10^{-3}$ lbf s/ft$^2$, flows steadily down a surface inclined $\theta = 30^\circ$ below the horizontal in a film of thickness $h = 0.125$ in. The velocity profile is given by

$$ u = \frac{\rho g}{\mu} \left( hy - \frac{y^2}{2} \right) \sin \theta $$

(Coordinate $x$ is along the surface and $y$ is normal to the surface.) Plot the velocity profile.

Determine the magnitude and direction of the shear stress $\tau$ that acts on the surface.

**Given:**

- $\mu = 2.15 \times 10^{-3}$ lbf s/ft$^2$
- $h = 0.125$ in
  - $= \frac{0.125}{12}$ ft
- $\rho_{water} = 1.94$ slugs/ft$^3$

$$ u = \frac{\rho g}{\mu} \left( hy - \frac{y^2}{2} \right) \sin \theta $$

$$ = \frac{(0.85 \times 1.94) \times 32.14}{2.15 \times 10^{-3}} \left[ \frac{0.125}{12} y - \frac{y^2}{2} \right] \sin 30^\circ $$

$$ u = u(y) = 24850 \left[ 0.010417 y - 0.5y^2 \right] \times 0.5 $$

$$ = 124.43 y - 621.25 y^2 $$

$$ \tau_w = \mu \left[ \frac{du}{dy} \right]_{y=0} = \mu \frac{du}{dy} \bigg|_{y=0} $$

$$ = \mu [124.43 - 124.05 y] y = 0 $$

$$ = \mu \times 124.43 = 2.15 \times 10^{-3} \times 124.43 $$

$$ \tau_w = 0.27827 \text{ lbf/ft}^2 \text{ in positive } x \text{-dir.} $$

**Ans.**

Take few values like $y = (0, 0.25h, 0.5h, 0.75h, h)$

and get the values. The profile will look something like shown in diagram.