MAE 2381: EXPERIMENTAL METHODS AND MEASUREMENTS

Fall 2005

FINAL EXAMINATION

December 7, 2005

8:00 — 10:30 a.m.

INSTRUCTIONS

• This is a closed-book/closed-notes examination. All formulas and constants will be given to you.
• This quiz is conducted in accordance with University rules regarding academic honesty.
• There is only one correct answer per question/problem. Two points for each correct answer. The total score is 50.
• You have 2½ HOURS.
• This booklet consists of fourteen (14) pages.
Do not fill

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Information you may need

- The error $w_y$ of a function $y = f(x_1, x_2, \ldots, x_n)$ where the errors of the independent parameters are $w_1, w_2, \ldots, w_n$ is given by
  
  $$w_y = \left[ \left( \frac{\partial y}{\partial x_1} w_1 \right)^2 + \cdots + \left( \frac{\partial y}{\partial x_i} w_i \right)^2 + \cdots + \left( \frac{\partial y}{\partial x_n} w_n \right)^2 \right]^{1/2}$$

- The range over which the possible values of the true mean value might lie at some probability level $P$ based on sampled values is given as
  
  $$\bar{x} \pm \frac{t_{v, P}}{\sqrt{N}} S (P\%)$$

- The total number of measurements $N_T$ needed to establish a certain precision interval CI is given iteratively by
  
  $$N_T = \left( \frac{t_{N_{i-1}, P}}{d} S_1 \right)^2 (P\%)$$

  where $d = CI/2$ and the subscript 1 refers to an initial test with a small sample.

<table>
<thead>
<tr>
<th>Confidence interval $z$</th>
<th>Confidence level %</th>
<th>Level of significance $\alpha$</th>
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<tbody>
<tr>
<td>3.30</td>
<td>99.9</td>
<td>0.001</td>
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<td>1.0</td>
<td>68.3</td>
<td>0.317</td>
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- A measurement is expressed as follows: $x = \bar{x} + \Delta$ (% confidence level), where $\bar{x}$ is the estimated mean and $\sigma$ is the estimated standard deviation. The confidence interval expresses the probability that the mean value will lie within a certain number of $\sigma$ values and is given by the symbol $z$ such that $\Delta = z\sigma / \sqrt{n}$ where $n$ is the sample size.
Gaussian probability and cumulative distributions
### Answer Key

**One-sided integral solutions for:**

\[
p(z) = \frac{1}{\sqrt{2\pi}} \int_{z}^{\infty} \exp\left(-\frac{\theta^2}{2}\right) d\theta
\]

<table>
<thead>
<tr>
<th>Z</th>
<th>0.0000</th>
<th>0.0040</th>
<th>0.0080</th>
<th>0.0120</th>
<th>0.0160</th>
<th>0.0199</th>
<th>0.0239</th>
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**Probability values for normal error function:**

- \[ \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp\left(-\frac{\theta^2}{2}\right) d\theta \]

---

*Note: The table continues with values for different Z values.*
The range over which the possible values of the true mean value might lie at some probability level $P$ based on sampled values is given as

$$\bar{x} \pm \frac{t_{\nu, P}}{\sqrt{N}} S (P\%)$$

The total number of measurements $N_T$ needed to establish a certain confidence interval CI is given iteratively by

$$N_T = \left( \frac{t_{N, -1, P}}{d} S_1 \right) ^2 (P\%),$$

where $d = CI/2$ and the subscript 1 refers to an initial test with a small sample.
## ANSWER KEY

Values for $\chi^2_a$

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$\chi^2_{0.99}$</th>
<th>$\chi^2_{0.975}$</th>
<th>$\chi^2_{0.95}$</th>
<th>$\chi^2_{0.90}$</th>
<th>$\chi^2_{0.50}$</th>
<th>$\chi^2_{0.05}$</th>
<th>$\chi^2_{0.025}$</th>
<th>$\chi^2_{0.01}$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.001</td>
<td>0.004</td>
<td>0.016</td>
<td>0.455</td>
<td>3.841</td>
<td>5.024</td>
<td>6.635</td>
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<td>2</td>
<td>0.020</td>
<td>0.051</td>
<td>0.103</td>
<td>0.211</td>
<td>1.386</td>
<td>5.991</td>
<td>7.378</td>
<td>9.210</td>
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<td>3</td>
<td>0.115</td>
<td>0.216</td>
<td>0.352</td>
<td>0.584</td>
<td>2.366</td>
<td>7.815</td>
<td>9.348</td>
<td>11.345</td>
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<td>4</td>
<td>0.297</td>
<td>0.484</td>
<td>0.711</td>
<td>1.064</td>
<td>3.357</td>
<td>9.488</td>
<td>11.143</td>
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<td>0.554</td>
<td>0.831</td>
<td>1.145</td>
<td>1.610</td>
<td>4.351</td>
<td>11.070</td>
<td>12.832</td>
<td>15.086</td>
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<td>0.872</td>
<td>1.237</td>
<td>1.635</td>
<td>2.204</td>
<td>5.348</td>
<td>12.592</td>
<td>14.449</td>
<td>16.812</td>
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<td>8</td>
<td>1.647</td>
<td>2.180</td>
<td>2.733</td>
<td>3.490</td>
<td>7.344</td>
<td>15.507</td>
<td>17.535</td>
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<td>16</td>
<td>5.812</td>
<td>6.908</td>
<td>7.962</td>
<td>9.312</td>
<td>15.338</td>
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<tr>
<td>60</td>
<td>37.485</td>
<td>40.482</td>
<td>43.188</td>
<td>46.459</td>
<td>59.335</td>
<td>79.082</td>
<td>83.298</td>
<td>88.379</td>
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</table>
For each problem, mark the most appropriate or correct answer in the corresponding box.

1. The zero-order uncertainty of an instrument is $u_0 = \pm \frac{1}{2}$ resolution (95%). The $\pm \frac{1}{2}$ resolution is
   a. Arbitrary
   b. Governed by state and federal regulations
   c. Required by the International Standards Organization
   d. None of a. – c
   e. All of a. – c

2. A digital display shows 4 digits. Such a display is known as a
   a. 3 digit display
   b. 3½ digit display
   c. 4 digit display
   d. 4½ digit display
   e. Cannot say for sure since it depends on the accuracy of the transducer

3. Only one of the following is MOST appropriate concerning uncertainty analysis
   a. It is used to determine the major independent parameters that govern an experiment
   b. It is a statistical evaluation of the error or uncertainty of a measurement
   c. It is used to determine if the methodology can be justified on theoretical grounds
   d. It is used to compare the results against those obtained elsewhere or against handbook data
   e. It is used to calibrate instruments that is affected by multiple inputs

4. One of the following is not a reason for bias to occur. Bias can occur due to the following reasons.
   a. Random fluctuations in the data
   b. Instrumentation is unable to resolve the data
   c. Human error
   d. Un-calibrated instrumentation
   e. Electromagnetic interference
5. In order to perform statistical analysis on time-series data, the data must
   a. Be sufficiently large in order to display a Gaussian behavior ✓
   b. Be filtered to prevent aliasing
   c. Be steady in the mean sense ✓
   d. Be sufficiently resolved
   e. Not be saturated

6. For a Gaussian distribution, \( \pm 2\sigma \) (2 TIMES the standard deviation) corresponds to a probability of approximately
   a. 25%
   b. 50%
   c. 68%
   d. 95% ✓
   e. 99%

7. Implicit in the use of the Gaussian (or normal) distribution is that
   a. There is a finite data set with a well-defined mean and standard deviation
   b. There is an infinite data set with a well-defined mean and standard deviation ✓
   c. One can ignore the mean and standard deviation
   d. The data can be fitted by any mathematical function with an arbitrary confidence level
   e. It allows bad data to be rejected with objective criteria

8. If a measurement is expected to follow the Gaussian distribution, one of the following CANNOT be done
   a. The mean and standard deviation can be computed
   b. The probability that a certain range of occurrences can be computed
   c. An upper bound can be computed given the probability for its occurrence
   d. A level of significance can be assigned to the mean, from which the minimum number of measurements that is needed to estimate the mean can be computed
   e. The bias of the data can be estimated ✓
9. One of the following is **NOT** true regarding regression analysis

   a. It assumes that the dependent variable \( (y \text{ say}) \) follows a Gaussian distribution about each fixed value of the independent variable \( (x \text{ say}) \)

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   b. It is commonly a polynomial fit but need not be so

   c. The term least squares is usually associated with regression analysis and it refers to minimization of the sum of the squares of the deviations between the actual data and the curve fit

   d. The type of fit chosen depends on the amount of data gathered ✓

   e. It establishes a functional relationship between the dependent variable and the independent variable which may or may not be physical

10. The Gaussian distribution is used as the basis to produce the statistics of many engineering processes. The **primary** reason for its widespread use is

   a. It is familiar

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   b. Most engineering processes require only a mean and a standard deviation for characterizing their statistical variations

   c. A result from the central limit theorem which states that an infinite sum of actual distributions would average to be a Gaussian distribution ✓

   d. Most of the important features of real statistical variations are captured by the Gaussian distribution

   e. It is the basis of other distributions such as the T and chi distributions

11. Strictly speaking, the observed mean value from a series of observations

   a. Is dependent on the confidence level assigned to the observation

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   b. Must undergo the Student T test to be valid

   c. Can only be made if the observations are expected to follow a Gaussian distribution

   d. Can only be valid if the large dataset is broken up into a number of smaller sets, with their individual means obtained and then averaged

   e. Is only an estimated value that approaches the (theoretical) true value as the number of observations approach infinity ✓

12. One of the following is **NOT** an implicit assumption made for performing an uncertainty analysis.

   a. The test objectives are known

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   b. Data are obtained under fixed operating conditions (steady or otherwise)

   c. The operators have some experience with the test hardware and procedures

   d. The experimental data can be fitted by a regression analysis ✓

   e. The measurement itself is a clearly defined process in which all known calibration corrections for bias have been applied
ANSWER KEY

13. The topic of infinite statistics refer to
   a. Statistical analysis based on the Gaussian distribution
   b. The assumption that the value of any statistical measure (mean, standard deviation, etc.) can range from $-\infty$ to $+\infty$
   c. The assumption that an “infinitely large” dataset is available for performing the statistics
   d. There is no bias in the data
   e. There is a curvefit which will yield a correlation coefficient of unity

14. Consider a probability distribution function $p(x)$. The probability of an event $x \leq \xi$ happening can be evaluated by
   a. $\Pr(x \leq \xi) = p(\xi)$
   b. $\Pr(x \leq \xi) = p(\xi) - p(-\infty)$
   c. $\Pr(x \leq \xi) = \frac{d}{dx} p(x = \xi)$
   d. $\Pr(x \leq \xi) = \int_{-\infty}^{\xi} p(x)dx$
   e. $\Pr(x \leq \xi) = \int_{0}^{\xi} p(x)dx$

15. The expression for the mean value of a distribution $\bar{x}$, based on the probability distribution function $p(x)$, is given by
   a. $\bar{x} = \int_{-\infty}^{\infty} x p(x)dx$
   b. $\bar{x} = \int_{-\infty}^{\infty} x p(x)dx$
   c. $\bar{x} = \int_{-\infty}^{\infty} x^2 p(x)dx$
   d. $\bar{x} = \int_{-\infty}^{\infty} \frac{p(x)}{x}dx$
   e. $\bar{x} = 2 \int_{0}^{\infty} x p(x)dx$
16. Only **ONE** of the processes below is considered to be as close to theoretically random as possible

   a. Daily traffic flow
   b. Growth of a seedling
   c. Flapping of a flag
   d. Passage of a tornado
   e. Rushing sound from an open car window  

17. It is a well-observed fact that measurements taken under seemingly identical conditions show variations in their measurements (as you have experienced in the laboratory sessions). One of the following is **NOT** a source for such variability

   a. Resolution of the data acquisition system
   b. Noise in the system
   c. Bias in the measurement system  
   d. Sample size
   e. Spatial and/or temporal variations of the measured variable  

18. One of the following is **INCORRECT** regarding design-stage uncertainty

   a. An intelligent guess (guesstimate) of the uncertainties of the hardware  
   b. A systematic approach for selecting instruments, measuring techniques and an approximate analysis of the uncertainty likely to be encountered
   c. There is no need to distinguish between bias and precision (also known as random) errors
   d. It requires an estimate of uncertainties of individual components and their combined contribution to the system uncertainty
   e. It will likely require the use of a manufacturer’s statement regarding instrument error  

19. One of the following is **INCORRECT** regarding bias errors

   a. Bias sometimes depends on experience
   b. Bias can only be estimated by comparison
   c. Bias can be characterized by a Gaussian distribution  
   d. Bias can sometimes be eliminated by calibration
   e. Bias may sometimes be detected by performing the experiment using different instruments
20. Most controlled laboratory experiments involve so-called stationary processes. Which ONE of the following statement is true
   a. The mean can be estimated
   b. A polynomial fit can be performed on the data
   c. The design-stage uncertainty can be estimated
   d. The data can be represented as a Gaussian distribution
   e. Higher-order statistics such as skew and kurtosis are meaningless

21. A purpose of calibration is to reduce measurement system errors. ONE of the following is not a potential sources of calibration errors include
   a. Calibration technique
   b. Primary to inter-laboratory standard
   c. Laboratory standard to measurement system
   d. Transfer to laboratory standard
   e. Digitizer resolution

22. One of the following is NOT associated with data acquisition errors.
   a. Operating conditions
   b. Sensor installation
   c. Environmental effects
   d. Electromagnetic interference
   e. Signal conditioning limitations

23. Mathematically, uncertainty estimation can be considered to be a problem in
   a. Stochastics
   b. Sequential perturbation
   c. Parallel perturbation
   d. Least-squares regression
   e. Fourier analysis
24. It is not possible to determine the true value in experimental work. Instead, we use the term uncertainty, which can be regarded as

a. An estimate of the probable error of the measurement ✓

b. Departure from the Gaussian distribution

c. An interval of one standard deviation from the mean

d. An interval of at least three standard deviations from the mean

e. A lack of agreed standards on the measurement procedure

25. The power dissipated by a resistor is given by \( P = \frac{V^2}{R} \), where \( R = 1000 \ \Omega \pm 5\% \) and \( V = 6 \pm 0.2 \ \text{V} \). What is the uncertainty in the power measurement?

a. 1 % ✓

b. 3 %

c. 5 %

d. 7 %

e. 8 %

\[
\frac{\partial P}{\partial V} = \frac{2V}{R} = \frac{2 \times 6}{1000} = 0.012 \ \text{V/\Omega} \\
\frac{\partial P}{\partial R} = -\left(\frac{V}{R}\right)^2 = -12 \times 10^{-6} \ \text{V}^2/\Omega^2 \\
\]

\[
u_P = \sqrt{\left(\frac{\partial P}{\partial V} u_V\right)^2 + \left(\frac{\partial P}{\partial R} u_R\right)^2} = \sqrt{\left(0.012 \times 0.2\right)^2 + \left(-12 \times 10^{-6} \times 0.05 \times 1000\right)^2} \\
= \sqrt{0.0025 + 0.00005} \\
= 0.05 \ \text{W} \\
= 7\% 
\]