1. A 0.45-kg soccer ball is stationary just before it is kicked upward at 12 m/sec. If the impact lasts 0.02 sec, what average force is exerted on the ball by the player's foot.

\[ v_0 = 0 \]

\[ v_1 = 12 \text{ m/sec} \]

\[ m = 0.45 \text{ kg} \]

\[ y: \int F \, dt = m v_1 - m v_0 \]

\[ F \Delta t = 0.45(12) = 5.40 \text{ N} \cdot \text{s} \]

\[ \Delta t = 0.02 \]

\[ F = 270 \text{ N} \]
1. Two cars with energy-absorbing bumpers collide head-on. Their speeds are \( v_A = 3 \text{ m/sec} \) and \( v_B = 2 \text{ m/sec} \), respectively. The weights of the cars are \( W_A = 2800 \text{ lb} \) and \( W_B = 4400 \text{ lb} \). The coefficient of restitution of the impact \( e = 0.2 \). What are the velocities of the cars immediately after the collision.

\[
M_A v_A + m_B v_B = M_A v'_A + m_B v'_B
\]

\[
\frac{2800 \times 3}{32.17} + \frac{4400 \times (-2)}{32.17} = \frac{2800}{32.17} v'_A + \frac{4400}{32.17} v'_B
\]

\[
-400 = 2800 v'_A + 4400 v'_B
\]

\[
e = \frac{v'_B - v'_A}{v_A - v_B} = 0.2 = \frac{v'_B - v'_A}{3 - (-2)}
\]

\[
v'_B - v'_A = 1
\]

\[
v'_B = 0.333 \text{ mph}
\]

\[
v'_A = -0.667 \text{ mph} \quad \text{or} \quad 0.667 \text{ mph}
\]
2. For the figure shown, the crank AB has a constant clockwise angular velocity of 200 rpm (revolutions per minute). Determine the velocity of the piston P.

\[ \omega_{AB} = 20.944 \, \text{rad/sec} \]

\[ V_B = 59.23 \, \text{in/sec} \]

\[ V_B = V_{B-IC} \omega_{BP} \]

From Law of Sines

\[ \frac{6.325}{\sin 45^\circ} = \frac{r_{B-IC}}{\sin 63.43^\circ} \]

\[ \frac{r_{B-IC}}{\sin 71.57^\circ} = \frac{8.487 \, \text{in}}{\sin 45^\circ} \]

\[ r_{B-IC} = 8.487 \, \text{in} \quad r_{P-IC} = 8 \, \text{in} \]

\[ \omega_{BP} = \frac{59.23}{8.487} = 6.98 \, \text{rad/sec} \]

\[ V_P = V_{P-IC} \omega_{BP} \]

\[ = 8(6.98) \]

\[ = 55.83 \, \text{in/sec} \]

\[ = 4.65 \, \text{ft/sec} \]
2. For the figure shown, the crank AB has a constant clockwise angular velocity of 200 rpm (revolutions per minute). Determine the velocity of the piston P.

\[ \omega_{AB} = 200 \text{ rev/min} = 20.944 \text{ rad/sec} \]

\[ V_B = 2.828 \left(20.944 \right) \]

\[ = 59.23 \text{ in/sec} \]

\[ \bar{V} = V_B + \bar{V}_p \]

\[ V_p \bar{i} = 59.2 \left[ \cos 45^\circ \bar{i} - \sin 45^\circ \bar{j} \right] \]

\[ + 6.325 \omega_{bp} \left[ \sin \phi \bar{i} + \cos \phi \bar{j} \right] \]

\[ X: \quad V_p = 41.875 + 2\omega_{bp} \]

\[ Y: \quad 0 = -41.875 + 6\omega_{bp} \]

\[ \omega_{bp} = 6.98 \text{ rad/sec} \]

\[ V_p = 55.81 \text{ in/sec} \]
2. For the figure shown, the crank AB is rotating at 2000 rpm (revolutions per minute). Determine the velocity of the piston C at the instant shown.

\[ \omega_{AB} = 2000 \frac{\text{rev}}{\text{min}} = 209 \frac{\text{rad}}{\text{sec}} \]

\[ \phi = \tan^{-1} \frac{0.05}{0.175} = 15.95^\circ \]

\[ V_B = (0.0707) 209 = 147.76 \frac{\text{m}}{\text{sec}} \quad \omega_{AB} \]

\[ \vec{V}_c = \vec{V}_B + \vec{V}_{cb} \]

\[ V_c \hat{i} = 147.76 \left[ \cos 45^\circ \hat{i} - \sin 45^\circ \hat{j} \right] + 0.182 \omega_{bc} \left[ \sin \phi \hat{i} + \cos \phi \hat{j} \right] \]

\[ x: \quad V_c = 10.34 + 0.05 \omega_{bc} \]

\[ y: \quad 0 = -10.34 + 0.175 \omega_{bc} \]

\[ \omega_{bc} = 59.1 \frac{\text{rad}}{\text{sec}} \quad V_c = 13.3 \frac{\text{m}}{\text{sec}} \]
3. A disk rolls on a curved surface as shown. The bar rotates at 10 rad/sec in the CCW (counterclockwise) direction. Determine the velocity of point A.

\[ V_c = 10 \text{ rad/sec} \times 1.12 \]
\[ = 1.2 \text{ m/sec} \]
\[ = 0.04 \omega_c \quad \omega_c = 30 \text{ rad/sec} \]

\[ V_A = r_{OA} \omega_c = 0.565 \times 30 \]
\[ = 1.697 \text{ m/sec} \]

\[ \theta \text{ at } 45^\circ \]

\[ \text{Diagram showing circular motion} \]
3. A disk rolls on a curved surface as shown. The bar rotates at 10 rad/sec in the CCW (counterclockwise) direction. Determine the velocity of point A.

\[
V_c = 10 \text{ rad/sec} \times 0.12 = 1.2 \text{ m/sec}
\]

\[
= 0.04 \omega_c \quad \omega_c = 30 \text{ rad/sec}
\]

\[
\overrightarrow{V_A} = \overrightarrow{V_c} + \overrightarrow{V_{A/c}}
\]

\[
V_{A_x} \hat{i} + V_{A_y} \hat{j} = 1.2 \hat{j} + 0.04 \omega_c \hat{i}
\]

\[
\begin{align*}
\text{X:} & \quad V_{A_x} = 0.04(30) = 1.2 \text{ m/sec} \\
\text{Y:} & \quad V_{A_y} = 1.2 \text{ m/sec}
\end{align*}
\]

\[
\overrightarrow{V_A} = \sqrt{(1.2)^2 + 1.2^2} = 1.7 \text{ m/sec} \quad \angle 45^\circ
\]
3. For the figure shown, the diameter of the disk is 1 m, and the length of the bar AB is also 1 m. The disk is rolling to the left, and point B slides on a plane smooth surface. Determine the angular velocity of bar AB and the velocity of point B.

\[ V_A = 7.07(4) = 2.828 \, \text{m/s} \]

\[ V_B = V_A + V_{BA} \]

\[ V_B = 2.828 \left[ -\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j} \right] + (1) \omega_{AB} \left[ \sin 30^\circ \hat{i} + \cos 30^\circ \hat{j} \right] \]

\[ X: \quad V_B = -2 + 5 \omega_{AB} \]

\[ Y: \quad 0 = 2 + 8.66 \omega_{AB} \]

\[ \omega_{AB} = -2.31 \, \text{rad/s} \quad V_B = -3.155 \, \text{m/s} \]
4. A boy standing at a distance of \( d = 4 \text{ m} \) from the bottom of a building throws a ball against the wall. He releases the ball at a velocity of 15 m/sec at an angle of 30° with respect to the horizontal. The coefficient of restitution \( e = 0.40 \). Determine the velocity of the ball immediately after it bounces off the wall.

\[
\begin{align*}
\dot{a}_y &= -9.81 \text{ m/s}^2 = \frac{\text{d}v_y}{\text{d}t} \\
v_x &= v_{x_0} = 15 \cos 30° = 13 \text{ m/s} \\
\vec{v} &= 13.0 \, \hat{i} + (17.50 - 9.81t) \, \hat{j} \\
\vec{r} &= 13.0t \, \hat{i} + (7.5t - 4.905t^2) \, \hat{j}
\end{align*}
\]

Ball strikes the wall at \( x = 4 \text{ m} \)

\[
\begin{align*}
4 &= 13.0t \\
t &= 0.308 \text{ sec} \\
\vec{v} &= 13 \, \hat{i} + 4.48 \, \hat{j} \\
v_{x_y} &= v_{f_y} = 4.48 \text{ m/s} \\
e &= 0.4 = -\frac{v_{f_x} - 0}{13 - 0} \\
v_{f_x} &= 5.20 \text{ m/s}
\end{align*}
\]

After Impact:

\[
\begin{align*}
v_f &= v_{f_x} \, \hat{i} + v_{f_y} \, \hat{j} = -5.20 \, \hat{i} + 4.48 \, \hat{j} \\
&= 6.86 \text{ m/sec}
\end{align*}
\]
4. A baseball bat strikes a baseball as shown. The ball is traveling horizontally to the right with a velocity of 132 ft/sec before impact. The velocity of the bat, also traveling horizontally, but to the left, remains constant at 60 ft/sec (no change before and after the impact). The coefficient of restitution as a consequence of the collision \( e = 0.2 \). Determine the velocity of the ball after impact.

\[
V_{\text{ball}, n_i} = 132 \cos 45^\circ = 93.34 \, \text{ft/s}
\]

\[
V_{\text{bat}, n_i} = -60 \cos 45^\circ = -42.43 \, \text{ft/s}
\]

\[
V_{\text{ball}, n_i} = V_{\text{bat}, n_i}
\]

\[
e = 0.2 = \frac{V_{\text{ball}} - (-42.43)}{93.34 - (-42.43)}
\]

\[
V_{\text{ball}, f} = -69.58 \, \text{ft/sec}
\]

\[
V_{\text{bat}, f} = 93.34 \, \text{ft/sec}
\]

\[
V_{\text{ball}, f} = \sqrt{(-69.58)^2 + (93.34)^2} = 116.42 \, \text{ft/sec} \approx 81.47^\circ
\]
5. A satellite is in an elliptical orbit about the earth with distances as shown. Use the conservation of angular momentum and energy to determine the radial velocity, \( v_r \), and the transverse velocity \( v_\theta \) of the satellite at B.

\[
\mathbf{H}_0 = \mathbf{r}_A \times \mathbf{v}_A = \mathbf{r}_B \times \mathbf{v}_B \sin \theta
\]

\[
= 8000 \times (8640)
\]

\[
= 16,038 \, v_B \sin 60.08^\circ
\]

\[
v_B = 4972.66 \, \text{m/sec}
\]