1. As an airplane begins its takeoff run, the normal forces exerted on the tires by the runway at A and B are $N_A = 720 \text{ lb}$ and $N_B = 1600 \text{ lb}$. Determine the magnitude of the airplane's acceleration.

For the FBD

$$\sum F_y = N_A + N_B - W = 0$$

$$W = 720 + 1600 = 2320 \text{ lb}$$

$$\sum F_x = T = ma$$

$$\sum M = 2N_B - 5N_A + (1) T = 0$$

$$T = 3600 - 3200 = 400 \text{ lb}$$

$$T = 400 = ma = \frac{2320}{32.17} \text{ slugs, } a$$

$$a = 5.546 \text{ ft/sec}$$
1. The total weight of the go-cart and driver is 240 lbs. The location of the combined mass center is shown. As a consequence of friction, the two wheels in the back exert a 24-lb horizontal force on the track. Neglect any horizontal forces exerted by the front tires and determine a) the magnitude of the go-cart's acceleration, and b) the normal forces exerted on the tires at A and B.

\[ \sum F_x = 24 = \frac{240}{32.17} \ a \]
\[ a = 3.22 \ \text{ft/s}^2 \]
\[ \sum M_G = 16 A - 448 - 24(15) = 0 \]
\[ + \sum F_y = A + B - 240 = 0 \]
\[ A = 182 \ \text{lb} \]
\[ B = 58 \ \text{lb} \]
2. For the figure below, a horizontal bar with a mass \( m = 10 \) kg and length of \( l = 8 \) meters is released from rest. The mass center of the bar is located at its center, but the bar rotates about a point \( x = 3 \) meters from the mass center. Determine a) the mass moment of inertia of the bar about \( O \), b) angular velocity of the bar about \( O \) when \( \theta = 90^\circ \), and b) the \( x \) and \( y \) components of the force on the pin at \( O \) when the bar is vertical (when \( \theta = 90^\circ \)).

Note: The moment about \( O \) is caused by gravity and varies with \( \theta \), or \( mgx \cos \theta \)

\[
\sum M_O = mgx \cos \theta = I_O \alpha
\]

\[
I_O = \frac{1}{12} mL^2 + x^2 m
\]

\[
= \frac{1}{12}(10)(8)^2 + (3)^2(10) = 143.33 \text{ kg m}^2
\]

\[
10(9.81)(3) \cos \theta = 143.33 \alpha
\]

\[
\alpha = 2.053 \cos \theta = \frac{d\omega}{d\theta}
\]

\[
\int_0^\omega \omega d\omega = 2.053 \int_0^\theta \cos \theta d\theta
\]

\[
\frac{\omega^2}{2} = 2.053 \sin \frac{\theta}{2}
\]

\[
\omega = 2.02 \text{ rad/sec}
\]

\[
\sum F_x = 0_x = m \alpha \cos \theta = m \alpha = m r \alpha = m r (2.053) \cos \theta = 0
\]

\[
\sum F_y = 0_y = (10) 9.81 = 10^\alpha \alpha = 10(3)(2.02)
\]

\[
0_y = 221.3 \text{ N}
\]
2. For the figure below, a horizontal bar with a mass \( m = 10 \, \text{kg} \) and length of \( l = 8 \, \text{meters} \) is released from rest. The mass center of the bar is located at its center, but the bar rotates about a point \( x = 3 \, \text{meters} \) from the mass center. Determine a) the mass moment of inertia of the bar about \( O \), b) angular velocity of the bar about \( O \) when \( \theta = 90^\circ \), and b) the \( x \) and \( y \) components of the force on the pin at \( O \) when the bar is vertical (when \( \theta = 90^\circ \)).

Note: The moment about \( O \) is caused by gravity and varies with \( \theta \), or \( mg \, x \, \cos \theta \)

\[
\sum T = T_1 + V_1 + V_{1-2} = T_2 + V_2
\]

\[
\text{O + O} + \text{O} = \frac{1}{2} I_0 \omega^2
- 3(10)(9.81)
\]

\[
I_0 = \frac{l}{12} ml^2 + 3m = 143.33 \, \text{kg} \, \text{m}^2
\]

\[
\omega = \frac{1}{2} (143.33) \omega_0^2 = 294.3
\]

\[
\omega_0 = 2.02 \, \text{r/sec}
\]

\[
\sum F_x = O_x = m a_{gx} = m r \alpha = m r (2.053) \cos \frac{\pi}{2} = 0
\]

\[
\sum F_y = O_y - 10(9.81) = m a_{gy} = 10(3)(2.02)^2
\]

\[
O_y = 221.3 \, \text{N}
\]
3. A bowling ball with a radius of 4.3 inches is released from rest on a 28° incline and rolls down the incline without slipping. Use the method of conservation of energy to determine the angular velocity of the ball when it has rolled 10 feet down the incline. Also, using kinematics of the ball, determine its acceleration and the time for the ball to move down the incline a distance of 10 feet. Note: The distance moved down the incline = r θ, and α = ω \frac{dω}{dθ}.

\[ T_1 + V_1 + U_{1-2} = T_2 + V_2 \]
\[ T_1 = 0 \quad V_1 = 0 \quad U_{1-2} = 0 \]
\[ T_2 = \frac{f}{2} m v^2 + \frac{f}{2} I ω^2 = \frac{7}{10} m v^2 \]
\[ V_2 = -m g s \sin 28° \]

\[ 0 = \frac{7}{10} m v^2 - m g (10) \sin 28° \]
\[ s = r θ = 10 \ t = \frac{4.3}{12} \ θ \]
\[ θ = 27.91 \ rad \]

\[ v = \frac{100g}{7} s \sin 28° = 215.78 \frac{ft}{s} \]
\[ v = 14.689 = \frac{4.3}{12} ω \]
\[ a = 41 \ \frac{rad}{sec} \]

\[ α = \frac{ω dω}{dθ} \]
\[ \int α dθ = \int ω dω = \frac{ω^2}{2} \]
\[ α(27.91) = (\frac{41 \ \frac{rad}{sec}}{12})^2 \]
\[ \frac{b) \ α = 30.11 \ \frac{rad}{sec} \]

\[ v = α t = 10.789 t = \frac{4.3}{12} (41 \ \frac{rad}{sec}) \]
\[ t = 1.36 \ sec \]
\[ α = 10.79 \ \frac{sec}{r} \]
2. A 24-lb uniform disk is placed in contact with an inclined surface as shown. A constant moment \( M = 8.5 \text{ ft lb} \) is then applied. The kinetic coefficient of friction between the disk and incline \( \mu_k = 0.3 \). The weight of the link AB is negligible. Determine a) the angular acceleration of the disk and b) the force in the link AB.

\[
\frac{I}{I_k} = \frac{1}{2} \left( \frac{24}{32.17} \right) \left( \frac{9}{12} \right)^2 = 0.2098 \text{ ft lb}^2
\]

\[
\sum M_k = 8.5 - 0.3N \left( \frac{9}{12} \right) = 0.2098 \alpha
\]

\[
\sum F_y = 0.3N \cos 30^\circ + N \cos 60^\circ - 24 = 0
\]

\[
0.2598N + 0.5N = 24
\]

\[
N = 31.587 \text{ lb}
\]

\[
\alpha = 6.639 \text{ rad/s}^2
\]

\[
\sum F_x = AB - 0.866N + 0.3N(0.5) = 0
\]

\[
AB = 22.616 \text{ lb}
\]
3. The 30-kg slender rod shown has a uniform cross section. Determine the acceleration of the mass center and the components of the forces at A immediately after the cord is cut. Assume that the end of the bar at A does not slide on the floor.

Note: The initial angular velocity $\omega$ is zero.

$$\sum M_A = 30(9.81) \cdot 5 \cos 60^\circ$$

$$= \frac{1}{3}(30)(10)^2 \alpha = 1000 \alpha$$

$$\alpha = 0.735 \text{ rad/sec}^2$$

$$\sum F_x = A_x = 30 q_{Ax}$$

$$\sum F_y = A_y - 30(9.81) = 30 q_{Ay}$$

$$a_{Ax} = 5 \alpha (\cos 30^\circ)$$

$$= 3.18 \text{ m/sec}^2$$

$$q_{Ay} = 5 \alpha \sin 30^\circ = 1.825 \text{ m/sec}^2$$

$$A_x = 30 (3.18) = 95.4 \text{ N}$$

$$A_y = 30(1.825) + 30(9.81) = 349.05 \text{ N}$$
4. A 45-kg cylinder is placed on a 25° inclined surface as shown. Starting from rest the cylinder rolls without slipping. The diameter of the cylinder is 300 mm. Use the energy method to determine a) the angular velocity \( \omega \) and b) the angular acceleration \( \alpha \) after the cylinder rolls 10 meters down the incline.

\[
T_1 + V_1 + U_{1-2} = T_2 + V_2
\]

\[
0 + 0 + 0 = \frac{1}{2} mV_c^2 + \frac{1}{2} I_c \omega^2
\]

\[
= 45(9.81)10 \sin 25°
\]

\[
\frac{1}{2} mV_c^2 + \frac{1}{2} \left( \frac{1}{2} m r^2 \right) \left( \frac{V_c}{r} \right)^2 = \frac{3}{4} mV_c^2
\]

\[
= 33.75 V_c^2
\]

\[
0 = 33.75 V_c^2 + 1865.65
\]

\[
V_c = \frac{7.43 \text{ m/sec}}{\text{a) } \omega = \frac{V}{r} = \frac{7.43}{.15} = 49.53 \text{ r/sec}}
\]

\[
\sum F = 45(9.81) \sin 25° - F = 45 \alpha = 45(.15)\alpha
\]

\[
F = 186.56 - 6.75 \alpha
\]

\[
(\sum M_c = .15 F = I \alpha = \frac{1}{2} (45)(.15)^2 \alpha = 0.506 \alpha)
\]

b) \[
F = 3.375 \alpha \quad \alpha = 18.43 \text{ r/s}^2
\]
4. A 16-lb disk is rotating about pin A in the vertical plane with an angular velocity $\omega = 3$ rad/sec when $\theta = 0^\circ$. Determine a) the angular velocity at the instant $\theta = 90^\circ$, and b) the horizontal and vertical components of the reaction at A when $\theta = 90^\circ$.

\[ I_A = \frac{1}{2} mr^2 + m \frac{c^2}{2} \]

\[ = \frac{1}{2} \left( \frac{16}{32.17} \right) (0.5)^2 + \left( \frac{16}{32.17} \right) (0.5)^2 \]

\[ = 0.1865 \text{ slug ft}^2 \]

\[ T_1 + V_1 + U_{1-2} = T_2 + V_2 \]

\[ \frac{1}{2} I \omega_1^2 + 0 + 0 = \frac{1}{2} I \omega_2^2 - mqr \]

\[ \frac{1}{2} (0.1865) (3)^2 = \frac{1}{2} (0.1865) \omega_2^2 - 16 (0.5) \]

\[ 0.839 = 0.932 \omega_2^2 - 8 \]

\[ \omega_2 = 9.738 \]

\[ \sum F_x = A_x = ma_{gx} = -\frac{16}{32.17} (1.5) \chi = 23.58 \text{ lb} \]

\[ \sum F_y = A_y - 16 = -\frac{16}{32.17} (0.7)^2 = -2.487 \chi \]

\[ (\sum M_A = \frac{5 (16)}{1.5} = (0.1865) \chi \chi = 42.9 \frac{V}{g_e} \]

\[ A_x = 23.58 \text{ lb} \]

\[ A_y = 5.33 \text{ lb} \]

\[ \text{When } \theta = 90^\circ \]