1. For the linkage shown, the block C is moving downward at 4 ft/sec. Use the method of Instantaneous Centers (I.C. Method) to determine the angular velocities of members AB and BC, and the linear velocity (magnitude and direction) of a point on member BC midway between B and C.

\[ \vec{V}_C = \vec{V}_B + \vec{V}_{C/B} \]

\[ -4 \hat{j} = -2 \omega_{AB} \hat{j} \]

\[ + 3 \omega_{BC} \left[ \cos 60^\circ \hat{i} - \sin 60^\circ \hat{j} \right] \]

\[ \chi: 0 = 3 \omega_{BC} \cos 60^\circ \quad \omega_{BC} = 0 \]

\[ \gamma: -4 = -2 \omega_{AB} - 0 \]

\[ \omega_{AB} = 2 \text{ rad/} \text{sec} \]

\[ V_G = 4 \text{ ft/} \text{sec} \]

\[ \overline{V}_C = 0 \]

\[ \omega_{BC} = 0 \]

All points \( V = 4 \text{ ft/} \text{sec} \)
1. For the linkage shown, the velocity of the slider block C is 4 ft/sec up the inclined groove. Use the method of Instantaneous Centers (I.C. method) to determine the angular velocities of links AB and BC, and the linear velocity (magnitude and direction) of a point on member BC midway between B and C.

\[ V_C = \sqrt{2} \omega_{bc} = 4 \text{ ft/s} \]
\[ \omega_{bc} = 2.83 \text{ rad/sec} \]
\[ V_B = (1)(2.83) = 2.83 \text{ ft/s} \]
\[ \omega_{AB} = 2.83 \text{ rad/sec} \]
\[ V_m = \frac{V_C}{2} \omega_{bc} = 1.118 (2.83) \]
\[ = 3.16 \text{ ft/s at } 26.5^\circ \]
1. For the figure shown, bar AB is attached to a collar that slides on a smooth shaft. The other end of the bar is attached to the slider. At the instant shown, the slider B moves to the right at 1.2 in/sec. For this instant, a) locate the IC for the bar AB, b) determine the angular velocity of bar AB, and c) determine the velocity of the collar at A.

\[ \mathbf{V}_B = \mathbf{V}_C + \mathbf{V}_{B/C} \]

\[ 1.2 = 0 + l_{BC} \omega_{AB} \]

From law of sines

\[ l_{BC} \sin 45^\circ = 24 \sin 15^\circ \]

\[ l_{BC} = 8.784'' \]

\[ \omega_{AB} = \frac{1.2}{8.784} = 0.1366 \text{ rad/Sec} \]

\[ \mathbf{V}_A = \mathbf{V}_C + \mathbf{V}_{A/C} \]

\[ = 0 + l_{CA} \omega_{AB} \]

\[ l_{AC} \sin 45^\circ = 24 \sin 120^\circ \]

\[ l_{AC} = 29.39'' \]

\[ V_A = 29.39 \times 13.41 = 4.015 \text{ in/sec} \]

\[ 45^\circ \]
2. The 4-inch radius wheel rolls without slipping to the left with a velocity of 50 in/sec. The distance from A to D is 3.0 inches. For the condition when $\beta = 0^\circ$, use the instantaneous center (I.C.) method to determine the following.

a) Angular velocity of the wheel, $\omega_D$,

b) Velocity of the connecting rod at point A,

c) Angular velocity of member AB, $\omega_{AB}$, and

d) velocity of the collar at B.

\[
V_D = 50 \text{ in/sec} = 4 \omega_D
\]

\[
\omega_D = 12.5 \text{ rad/sec}
\]

\[
V_A = 1 \cdot (12.5) = 12.5 \text{ in/sec}
\]

I.C. is at the contact of the wheel.

I.C. for member AB $= \infty$

\[
\therefore V_B = 12.5 \text{ in/sec}
\]

$\omega_{AB} = 0$

\[
V_D = 50 \text{ in/sec}
\]

\[
V_A = 12.5 \text{ in/sec}
\]
2. The slender rod AB slides on a smooth fixed half-cylindrical surface as shown. Point A moves to the right at a constant velocity \( v_A = 3 \text{ in/sec} \). The radius of the cylinder \( r = 10 \text{ inches} \), and the angle \( \theta = 30^\circ \). Use the vector method to determine the angular velocity \( \omega_{AB} \) of the rod at this instant of motion.

\[
\text{Use I.C. Method}
\]

\[
A_{IC} \cos 60^\circ = 17.32
\]

\[
A_{IC} = 34.64''
\]

\[
\tan 60^\circ = \frac{C_{IC}}{17.32}
\]

\[
C_{IC} = 30''
\]

\[
V_A = 3 \text{ in/sec} = A_{IC} \omega = 34.64 \omega
\]

\[
\omega_{AB} = 0.866 \text{ rad/sec}
\]

\[
V_c = C_{IC} \omega = 30 (0.866) = 26 \text{ m/sec}
\]
2. For the mechanism shown, the crank arm AB rotates at $\dot{\theta} = 7$ rev/min and is constant. For the instant shown, $\theta = 40^\circ$. Determine the angular velocity of the arm BC, $\omega_{BC}$, and the velocity of the drum center $V_c$.

\[
\omega_{AB} = 7 \text{ rev/min} \left( \frac{2\pi \text{ rad}}{\text{rev}} \right) \frac{\text{min}}{60 \text{ sec}} \\
= 7.733 \text{ rad/sec}
\]

\[
\sin \phi = \frac{\sin 40^\circ}{18} \quad \phi = 10.28^\circ
\]

\[
\vec{V}_b = \vec{V}_A + \vec{V}_{b/A} = \vec{V}_C + \vec{V}_{b/c}
\]

\[
0 + 5\omega_{AB} \left( -\sin 40^\circ \hat{i} + \cos 40^\circ \hat{j} \right)
\]

\[
= \vec{V}_C \hat{i} + 18\omega_{bc} \left( -\sin \phi \hat{i} + \cos \phi \hat{j} \right)
\]

\[
-2.356\hat{i} + 2.807\hat{j} = \vec{V}_C \hat{i} + 3.214\omega_{bc} \hat{i}
\]

\[
+ 17.71 \omega_{bc} \hat{j}
\]

\[
y: \quad 2.807 = 17.71 \omega_{bc} \quad \omega_{bc} = 0.158 \text{ rad/sec}
\]

\[
x: \quad -2.356 = V_c + 3.214 \times 0.158
\]

\[
V_c = 2.865 \text{ in/sec}
\]
For the piston linkage arrangement shown, crank AB rotates about a fixed point A with a constant angular velocity of 900 rev/min clockwise. When in the motion $\theta = 60^\circ$, determine the following.

a) Angular velocity of member BD, $\omega_{BD}$.
b) Linear velocity of the piston, $v_D$.
c) Angular acceleration of member BD, $\alpha_{BD}$

\[
\omega_{AB} = 900 \text{ rev/min} \left( \frac{2\pi}{60} \right) = 30\pi \text{ rad/sec} = 94.25 \text{ rad/sec}
\]

\[
\frac{\sin \phi}{2} = \frac{\sin 60^\circ}{6} \quad \phi = 16.78^\circ
\]

\[
\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}
\]

\[
\dot{\omega}_{D/B} = 16.4 \text{ rad/sec}^2 \quad \vec{v}_D = 191.64 \text{ ft/sec}
\]

\[
\vec{q}_D = \vec{q}_B + \vec{q}_{D/B}
\]

\[
\ddot{q}_0 = 2 (30\pi)^2 (-\cos 30^\circ \vec{i} - \sin 30^\circ \vec{j})
\]

\[
+ 6 (16.4)^2 (\sin \phi \vec{i} - \cos \phi \vec{j})
\]

\[
+ 6 \ddot{\alpha}_{D/B} (\cos \phi \vec{i} + \sin \phi \vec{j})
\]

\[
\ddot{\alpha}_{D/B} = 2597 \text{ rad/sec}^2
\]
3. For the figure shown, crank AB is rotating in the clockwise direction at a constant angular velocity of 1500 rpm (revolutions per minute). At the instant shown a) locate the instantaneous center of zero velocity (I.C.) for rod BC, b) use the I.C. location to determine the angular velocity of the rod BC, and c) determine the acceleration of the piston C at this instant.

\[ V_B = AB \omega_{AB} \quad \omega_{AB} = 1500 \text{ rpm} \]
\[ = 157.08 \text{ rad/sec} \]
\[ AB = \sqrt{50^2 + 50^2} \]
\[ = 70.71 \text{ mm} = 0.0707 \text{ m} \]
\[ V_B = 0.0707(157.08) = 11.106 \text{ m/sec} \]
\[ = 0.2475 w_{BC} \]

\[ w_{BC} = \frac{11.106}{0.2475} = 44.871 \text{ rad/sec} \]

\[ \alpha_{AB} = 0 \quad \bar{C} = \bar{A} + \bar{A}_{C/B} \]

\[ \bar{A}_{C/B} = -AB \omega_{AB}^2 \left[ \cos 45^\circ \hat{i} + \sin 45^\circ \hat{j} \right] \]
\[ + 1.182 \alpha_{BC} \left[ \sin \phi \hat{i} + \cos \phi \hat{j} \right] \]
\[ + 1.182 w_{BC}^2 \left[ -\cos \phi \hat{i} + \sin \phi \hat{j} \right] \]

\[ \alpha_{BC} = 7.624 \text{ rad/s}^2 \]

\[ \bar{A} = 1204 \text{ m/sec}^2 \]
3. The slider-crank mechanism shown has a crank arm length \( L_{AB} = 75 \text{ mm} \) and is connected to a rod of length \( L_{BC} = 175 \text{ mm} \). At the instant shown, \( \theta = 70^\circ \), and the crankshaft rotates at a constant rate of \( \omega_{AB} = \dot{\theta} = 4800 \text{ rev/min} \). Determine the angular velocity \( \dot{\phi} \) and the angular acceleration \( \ddot{\phi} \) of the connecting rod BC.

\[
\omega_{AB} = 160 \pi = 502.65 \text{ rad/s}
\]

\[
\overrightarrow{V}_B = \overrightarrow{V}_C + \overrightarrow{V}_{B/C}
\]

\[
-37.7 \cos 20^\circ \hat{i} + 37.7 \sin 20^\circ \hat{j}
\]

\[
= -V_c \hat{i} + 175 \dot{\phi} [\sin \phi \hat{i} + \cos \phi \hat{j}]
\]

\[
-35.43 \hat{i} + 12.89 \hat{j} = -V_c \hat{i} + 170 \dot{\phi} \hat{i} + 160 \ddot{\phi} \hat{j}
\]

\[
V_c = 12.89 = 160 \dot{\phi}
\]

\[
\ddot{\phi} = 80.56 \text{ rad/s}^2
\]

\[
\phi = 23.75^\circ
\]
3. The slider-crank mechanism shown has a crank arm length \( L_{AB} = 75 \, \text{mm} \) and is connected to a rod of length \( L_{BC} = 175 \, \text{mm} \). At the instant shown, \( \theta = 70^\circ \), and the crankshaft rotates at a constant rate of \( \omega_{AB} = \dot{\theta} = 4800 \, \text{rev/min} \). Determine the angular velocity \( \phi \) and the angular acceleration \( \ddot{\phi} \) of the connecting rod BC.

\[
\vec{a}_B = \vec{a}_c + \vec{a}_{B/c} \\
= -18,950 \cos 70^\circ \, \vec{i} - 18,950 \sin 70^\circ \, \vec{j} \\
= -a_c \vec{i} + 1135.7 \left[ \cos \phi \vec{i} - \sin \phi \vec{j} \right] \\
+ 1.75 \ddot{\phi} \left[ \sin \phi \vec{i} + \cos \phi \vec{j} \right] \\
\]

\[
= -6481.3 \, \vec{i} - 17,807 \, \vec{j} \\
= -a_c \vec{i} + 1039.5 \vec{i} \\
- 457.4 \, \vec{j} + 0.070 \ddot{\phi} \vec{i} \\
+ 0.16 \ddot{\phi} \vec{j} \\
\]

\[ \ddot{\gamma} = -17,807 = -457.4 + 0.16 \ddot{\phi} \]

\[ \ddot{\phi} = 108,435 \, \text{rad}^2/\text{sec}^2 \]