1. A boat is moving at 10 m/sec when its engine is shut down. Due to hydrodynamic drag, its subsequent acceleration is \( a = -0.05 \, v^2 \) (m/sec\(^2\)), where \( v \) is the velocity of the boat in m/sec. Determine the boat's velocity 4 seconds after the engine shuts down.

\[
Q = \frac{dv}{dt} = -0.05 \, v^2
\]

\[
\int_{10}^{v} \frac{dv}{v^2} = -0.05 \int_{0}^{t} dt = \int_{10}^{v} v^{-2} \, dv = -\frac{1}{v} \bigg|_{10}^{v}
\]

\[-\frac{1}{v} + \frac{1}{10} = -0.05 \, t\]

\[
\frac{1}{v} = \frac{1}{10} + 0.05 \, t = \frac{1 + 0.05 \, t}{10}
\]

\[
v = \frac{10}{1 + 0.05 \, t}
\]

For \( t = 4 \) sec

\[
v = 3.33 \, \text{m/sec}
\]
1. A car’s acceleration is related to its position by \( a = 0.01 s \text{ (m/sec}^2\text{)}. \) When \( s = 100 \text{ m} \), the car is moving at 12 m/sec. Determine how fast the car is moving when \( s = 420 \text{ m} \).

\[
\begin{align*}
\text{\(a\) } &= \frac{v}{ds} = 0.01s \\
\int_{12}^{v} v \, dv &= 0.01 \int_{100}^{420} s \, ds \\
\left. \frac{v^2}{2} \right|_{12}^{420} &= 0.01 \left. \frac{s^2}{2} \right|_{100}^{420} \\
\frac{v^2}{2} &= \frac{12^2}{2} + 0.01 \left( \frac{420^2 - 100^2}{2} \right) \\
V_f &= 42.5 \text{ m/sec}
\end{align*}
\]
1. The rocket sled shown starts from rest and accelerates at \( a = 30 + 2t \) (m/sec\(^2\)) until its velocity is 400 m/sec. It then hits a water brake and its acceleration is \( a = -0.003v^2 \) (m/sec\(^2\)) until its velocity decreases to 100 m/sec. Determine the distance the sled travels.

\[
\text{Acceleration Phase:} \quad a = 30 + 2t = \frac{dv}{dt}
\]

\[
\int dv = \int (30 + 2t) \, dt \quad \Rightarrow \quad v = 30t + \frac{2t^2}{2} = \frac{ds}{dt}
\]

\[
\int ds = \int (30t + t^2) \, dt \quad \Rightarrow \quad s = 15t^2 + \frac{t^3}{3}
\]

When \( v = 400 \text{ m/sec} \) acceleration ends, deceleration begins.

At this point, \( t = 10s \quad s = 1833 \text{ m} \)

For deceleration, \( a = v \frac{dv}{ds} = -0.003v^2 \)

\[
\int_{s_1}^{s_f} ds = -\frac{1}{0.003} \int_{v_i}^{v_f} \frac{vdv}{v^2}
\]

\[
s_f - 1833 = -\frac{1}{0.003} \left[ \ln (100) - \ln (400) \right]
\]

\[
s_f = 2300 \text{ m}
\]
2. A small marble rolls down a chute as shown described by the equation \( y = 1 - 0.5\sqrt{4 - x^2} \), where \( x \) and \( y \) are in feet. The speed of the marble is 12 ft/sec, and has a tangential acceleration \( \dot{v} = 8 \text{ ft/sec}^2 \) when it passes \( x_0 = 1.5 \text{ ft} \). Determine the total acceleration of the marble as it passes point A.

\[
\alpha = \sqrt{\alpha_t^2 + \alpha_n^2} \quad \alpha_t = \dot{v} = 8 \text{ ft/sec}^2
\]

\[
\alpha_n = \frac{V_t}{\rho} = \frac{144}{1.758} \text{ ft/sec}^2
\]

\[
\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}
\]

\[
y = 1 - 0.5(4 - x^2)^{1/2}
\]

\[
\frac{dy}{dx} = 0.5 \times (4 - x^2)^{-1/2}
\]

\[
\frac{d^2y}{dx^2} = 0.5(4 - x^2)^{-3/2} + 0.5x^2(4 - x^2)^{-3/2}
\]

At \( x = x_0 = 1.5 \text{ ft} \)

\[
\frac{dy}{dx} = 0.567 \quad \frac{d^2y}{dx^2} = 0.8639
\]

\[
\rho = 1.758 \text{ ft}
\]

\[
\alpha_n = \frac{144}{1.758} = 81.897 \text{ ft/sec}^2
\]

\[
\alpha = \sqrt{8^2 + 81.897^2} = 82.3 \text{ ft/sec}^2
\]
2. A small sphere slides along a rod described by the equation \( y = x^{\frac{1}{2}} \), where \( x \) and \( y \) are in meters. When the object is at point A (\( x = 2 \text{ m} \)), it is moving at \( v = 7 \text{ m/sec} \), but is slowing down at \( 2 \text{ m/sec}^2 \). Determine the acceleration of the sphere at this time.

\[
y = x^{\frac{1}{2}} \quad \frac{dy}{dx} = \frac{1}{2} x^{-\frac{1}{2}}
\]

\[
\frac{d^2 y}{dx^2} = -\frac{1}{4} (x^{-\frac{3}{2}})
\]

\[
\frac{1}{p} = \frac{d^2 y / dx^2} \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2} = 0.0741 \text{ m}^{-1}
\]

\( p = 13.5 \text{ m} \)

\( q_t = v = -2 \text{ m/sec} \quad q_n = \frac{v^2}{p} = \frac{7^2}{13.5} = 3.63 \text{ m/sec}^2 \)

\( q = \sqrt{q_t^2 + q_n^2} = 4.14 \text{ m/sec} \quad \phi = 80.61^\circ \)
2. A small sphere slides along a rod described by the equation $x^2 = 8y$, where $x$ and $y$ are in feet. When the object is at $x = -8$ ft, and $y = 8$ ft, it is moving at $v = 15$ ft/sec, but is slowing down at $3$ ft/sec$^2$. Determine the acceleration of the sphere at this time.

\[ y = \frac{x^2}{8} \]

\[ \frac{dy}{dx} = \frac{x}{4} \]

\[ \frac{d^2y}{dx^2} = \frac{1}{4} \]

\[ \frac{1}{P} = \frac{\frac{d^2y}{dx^2}}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} = \frac{\frac{1}{4}}{\left[1 + \left(\frac{x}{4}\right)^2\right]^{3/2}} \]

At $x = -8$, $y = 8$, $\frac{dy}{dx} = -2$  \[ \frac{1}{P} = 0.02236 \]

\[ P = 44.7214 \text{ ft} \]

\[ a_t = \dot{v} = -3 \text{ ft/s}^2 \]

\[ a_n = \frac{v^2}{P} = \frac{15^2}{44.7214} = 5.031 \text{ ft/s}^2 \]

\[ a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-3)^2 + (5.031)^2} = 5.86 \text{ ft/s}^2 \]

\[ \phi = \tan^{-1} \frac{dV}{dx} = \tan^{-1}(-2) = -63.43^\circ \]

\[ Q_t \quad Q_n \quad \theta \quad 30.81^\circ \quad 22.57^\circ \]

\[ \phi = 63.43^\circ \]
3. For the robot’s arm shown, determine the velocity of point P for $t = 0.8$ sec, where the radial and transverse positions are described by

$$r = 1 - 0.5 \cos (2\pi t) \quad \text{(meters)}$$
$$\theta = 0.5 - 0.2 \sin (2\pi t) \quad \text{(radians)}.$$

$$\dot{r} = \pi \sin (2\pi t)$$
$$\dot{\theta} = -0.4 \pi \cos (2\pi t)$$

For $t = 0.8$ sec

$$v_r = 0.845 \text{ m} \quad \dot{r} = -2.99 \text{ m/sec}$$
$$\dot{\theta} = -0.388 \text{ rad/sec}$$

$$V_r = \dot{r} = -2.99 \text{ m/sec} \quad V_\theta = r \dot{\theta} = -0.328 \text{ m/sec}$$

$$V = \sqrt{V_r^2 + V_\theta^2} = \sqrt{(-2.99)^2 + (-0.328)^2} = 3.0 \text{ m/sec}$$
3. The collar A slides on the circular bar as shown. The radial position of A in meters is given as a function of $\theta$ by $r = 2 \cos \theta$. At the instant shown, $\theta = 25^\circ$ and $d\theta/dt = 4 \text{ rad/sec}$. Determine the velocity of A in terms of polar coordinates.

$$r = 2 \cos \theta$$

$$r = -2 \sin \theta \quad \dot{\theta} = V_r$$

$$\dot{\theta} = 4 \text{ rad/sec}$$

$$V_\theta = r \dot{\theta} = (2) \cos 25^\circ (4) = 7.25 \text{ m/sec}$$

$$V_r = -3.381 \text{ m/sec}$$

$$V = \sqrt{V_r^2 + V_\theta^2} = \sqrt{(-3.381)^2 + (7.25)^2} = 8 \text{ m/sec}$$
3. A boat moves as shown at 4 knots and follows a path described by \( r = 10 \theta \) (m), where \( \theta \) is in radians. Determine the boat's velocity when \( \theta = 2\pi \) in terms of polar coordinates. Note: A knot is one nautical mile per hour, or 1852 m/hr.

\[
V = 4 \text{ knots} \left( \frac{1852 \text{ m}}{\text{hr}} \right) hr \frac{\text{hr}}{3600 \text{ sec}} = 2.06 \text{ m/sec}
\]

\[
v = 10 \theta \quad \dot{r} = V_r = 10 \dot{\theta}
\]

\[
V^2 = V_r^2 + V_\theta^2 = \dot{r}^2 + (r \dot{\theta})^2
\]

\[
= (2.06)^2
\]

\[
\text{At } \theta = 2\pi \quad r = 10(2\pi) = 62.8 \text{ m}
\]

\[
(2.06)^2 = (10 \dot{\theta})^2 + r^2 \dot{\theta}^2 = \dot{\theta}^2 [100 + 62.8^2]
\]

\[
\dot{\theta} = 0.0323 \text{ rad/sec}
\]

\[
V_r = 10 \dot{\theta} = 0.323 \text{ m/sec} \quad V_\theta = r \dot{\theta} = 2.032 \text{ m/sec}
\]

\[
V = \sqrt{V_r^2 + V_\theta^2} = \sqrt{(0.323)^2 + (2.032)^2} = 2.057 \text{ m/sec}
\]

\[
= 6.747 \text{ f/sec}
\]

\[
= 4.6 \text{ mi/hr} = 5.3 \text{ knots}
\]

**Note:** Circumference of the earth is 24,881 mi at the equator.

One minute of arc = \( \frac{24,881}{360(60)} = 1.1519 \text{ mi} = 1 \text{ Nautical mi}
\]

Knot = \( \frac{1 \text{ Nautical mi}}{hr} = \frac{1.1519 \text{ mi} (5280) ft + \text{ mi}}{hr \text{ mi} (3.28) ft} = 18.54 \text{ m/hr}
\]
4. Starring from rest, a Cheetah has a constant acceleration to a maximum velocity of 110 ft/sec. If the animal can reach its maximum speed in 4 seconds, and assuming it can hold its maximum speed thereafter, determine the total distance it can travel in 10 seconds.

\[ a = \frac{dv}{dt} \int a \, dt = \int dv \]

\[ 4a = 110 \quad a = 27.5 \, \text{ft/s}^2 = \frac{dv}{dt} \]

\[ \int dv = \int_{0}^{t} 27.5 \, dt \]

\[ v = 27.5 \, t = \frac{ds}{dt} \]

\[ \int_{0}^{4} v \, dt = \int_{0}^{4} 27.5 \, t \, dt = \int_{0}^{s} ds \]

\[ s_1 = \frac{27.5}{2} (4)^2 = 220 \, \text{ft} = 67.07 \, \text{m} \]

From 4 to 10 sec \[ s_2 = 110(6) = 660 \, \text{ft} \]

Total distance = \[ s_1 + s_2 = 880 \, \text{ft} = 268.29 \, \text{m} \]
4. At the instant shown, the horizontal component of acceleration of the 26,000-lb airplane due to the sum of external forces acting on it is 14 ft/sec². If the pilot suddenly increases the thrust \( T \) by 4000 lb in the direction of \( T \), determine the horizontal component of the plane's acceleration when the added thrust is applied.

Initially

\[
a = \frac{a_x}{\cos 15^\circ} = \frac{14 \text{ ft/s}^2}{\cos 15^\circ}
\]

\[
a = 14.494 \text{ ft/s}^2 \quad T = ma = \frac{26,000}{32.17} (14.494)
\]

\[
= 11,714 \text{ lb}
\]

For an increase in thrust of 4000 lb

\[
\bar{T} = 11,714 + 4000 = 15,714 \text{ lb} = \frac{26,000}{32.17} a_2
\]

\[
a_2 = 19.443 \text{ ft/s}^2
\]

\[
a_{2x} = 19.443 \cos 15^\circ = 18.78 \text{ ft/s}^2
\]
4. The lunar module descends to the moon's surface under retro-thrust as shown. At a height of 5 meters, it is traveling at 2 m/sec downward. At this point the engines are cut off abruptly, and the vehicle falls freely to the surface. If the lunar acceleration due to gravity is 1/6 that of earth, determine the impact velocity at which the vehicle will land.

\[
a = \frac{1}{6} g = v \frac{dv}{dz}
\]

\[
\int_{0}^{5} \frac{1}{6} g \, dz = \int_{2}^{v} v \, dv
\]

\[
\frac{1}{6} g (5 - 0) = \frac{v^2}{2} \bigg|_{2}^{v}
\]

\[
\frac{5(9.81)}{6} = \frac{1}{2} (v^2 - 4)
\]

\[
v^2 = 20.35 \quad v = 4.51 \text{ m/sec}
\]