Problem 17.5 (Page 387)

A solid is formed by revolving the shaded area completely around the y-axis as shown. Determine the radius of gyration, $k_y$ for this figure generated. The specific weight of the material $\gamma = 380 \text{ lb/ft}^3$.

\[
dI_y = \frac{1}{2} \, dm \, x^2 \\
= \frac{1}{2} \left( \pi x^2 \rho \, dy \right) \, x^2 \\
= \frac{1}{2} \pi \rho x^4 \, dy \\
= \frac{1}{2} \rho \pi \frac{y^3}{9} \, dy \\
I_y = \int dI_y = \frac{\rho \pi}{2(9)} \int_0^3 y^{12} \, dy \\
= 29.632 \rho
\]

\[
m = \int dm = \frac{1}{81} \rho \pi \int_0^3 y^6 \, dy = 12.117 \rho
\]

\[
k_y = \sqrt{\frac{I_y}{m}} = \sqrt{\frac{29.632 \rho}{12.117 \rho}} = 1.56 \text{ in.}
\]
Problem 17.14 (Page 389)

The assembly shown consists of a disk having a mass of 6 kg, and two slender rods, AB and DC, each having a mass of 2 kg/m. If $L = 0.75$ m, determine the moment of inertia of the assembly about an axis perpendicular to the screen and passing through $O$.

\[ I_o = I_{disk} + I_{AB} + I_{CD} \]

\[ I_{disk} = \frac{1}{2} b (b/2)^2 + b (1)^2 = 6.12 \text{ kg m}^2 \]

\[ I_{AB} = \frac{1}{12} \left[ \frac{2}{m_{AB}} \right] (1.3)^2 + 2 (1.3)(.15)^2 = 0.425 \text{ kg m}^2 \]

\[ I_{CD} = \frac{1}{12} \left[ \frac{2}{m_{CD}} \right] (.75)^2 + 2 (.75)(.15)^2 = 0.445 \text{ kg m}^2 \]

\[ I_o = 6.12 + 0.425 + 0.445 = 6.99 \text{ kg m}^2 \]

Total mass $= 6 + 2(1.3) + 2(.75) = 10.1$ Kg

\[ I_o = K^2 m = 6.99 = 10.1 K^2 \]

$K = 0.832$ m (Radius of Gyration)
Problem 17.54 (Page 414)

The 10-kg wheel shown has a radius of gyration \( k_A = 200 \text{ mm} \). If the wheel is subjected to a moment \( M = 5 \text{ t} \ \text{ (N} \cdot \text{m)} \), where \( t \) is in seconds, determine its angular velocity when \( t = 3 \text{ sec} \) starting from rest. Also, compute the reactions which the fixed pin \( A \) exerts on the wheel during the motion.

\[
\sum F_x = A_x = 0
\]

\[
\sum F_y = A_y - 10(9.81) = 0
\]

\[
A_y = 98.1 \text{ N}
\]

\[
\sum M_A = 5t = I_A \alpha
\]

\[
\alpha = \frac{d\omega}{dt} = 12.5 \text{ t}
\]

\[
\omega = \int_{0}^{3} 12.5t \, dt = \frac{12.5}{2} (3)^2
\]

\[
= 56.2 \text{ rad/sec}
\]

\[
I_A = 10(0.2)^2 = 0.4 \text{ Kg} \cdot \text{m}^2
\]
Problem 17.59 (Page 415)

A 10-lb bar is pinned at its center O as shown and is connected to a torsional spring. The spring has a stiffness $k = 5 \text{ ft-lb/rad}$, providing a torque $M = 5 \theta \text{ (ft-lb)}$, where $\theta$ is in radians. If the bar is released from rest when it is vertical at $\theta = 90^\circ$, determine its angular velocity at the instant $\theta = 0^\circ$.

\[
\sum M = -5\theta = \frac{1}{12} \left( \frac{10}{32.174} \right) (z)^2 \alpha
\]

\[
-48.3 \theta = \alpha = \omega \frac{dw}{d\theta}
\]

\[
-\int_{\pi/2}^{\theta} 48.3 \theta d\theta = \int_{0}^{\omega} \omega d\omega
\]

\[
48.3 \left( \frac{\pi}{2} \right)^2 = \frac{1}{2} \omega^2
\]

\[
\omega = 10.9 \text{ rad/sec}
\]
Problem 17.56 (Page 414)

The drum shown has a weight of 80 lbs with a radius of gyration \( k_0 = 0.4 \) ft. A cable is wrapped around the drum and is subjected to a vertical force \( P = 15 \) lbs. Determine the time needed to increase the drum's angular velocity from \( \omega_1 = 5 \text{ rad/sec} \) to \( \omega_2 = 25 \text{ rad/sec} \). Neglect the mass of the cable.

\[
\sum M = I \alpha
\]

\[
15(0.5) = \frac{80}{32.174} \cdot (0.4)^2 \alpha
\]

\[
\alpha = 18.87 \text{ rad/sec}^2 = \frac{d\omega}{dt}
\]

\[
\omega_2 = \omega_1 + \alpha t
\]

\[
25 = 5 + 18.87 t \quad t = 1.06 \text{ sec}
\]
Problem 17.58 (Page 415)

A cord is wrapped around the inner core of a spool. If the cord is pulled with a constant tension of 30 lbs and the spool is originally at rest, determine the spool's angular velocity when \( s = 8 \) ft of cord has unwound. Neglect the weight of the 8-ft portion of cord. The spool and the entire cord have a total weight of 400 lbs, and the radius of gyration about the axle is \( k_A = 1.3 \) ft.

\[
I_A = m k_A^2 = \frac{400}{32.2} (1.3)^2
\]

\[
= 20.99 \text{ slugs} \text{ ft}^2
\]

\[
\sum M = I_A \alpha
\]

\[
= 30(1.25) = 20.99 \alpha
\]

\[
\alpha = 1.786 \text{ r/s}^2
\]

\[
\theta = \frac{s}{r} = \frac{8}{1.25} = 6.4 \text{ rad}
\]

\[
\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)
\]

\[
= 0 + 2(1.786)(6.4 - 0)
\]

\[
\omega = 4.78 \text{ r/s}
\]
Problem 17.60 (Page 415)

A 10-lb bar is pinned at its center O as shown and is connected to a torsional spring. The spring has a stiffness \( k = 5 \) ft-lb/rad, providing a torque \( M = 5 \theta \) (ft-lb), where \( \theta \) is in radians. If the bar is released from rest when it is vertical at \( \theta = 90^\circ \), determine its angular velocity at the instant \( \theta = 45^\circ \).

\[
\begin{align*}
\sum M_o &= -5\theta = \frac{1}{12} \left( \frac{10}{32.17} \right) \theta^2 \\
\alpha &= -48.3 \theta = \frac{d\omega}{dt} = \omega \frac{d\omega}{d\theta} \\
\int_{\pi/2}^{\pi/4} -48.3 \theta \, d\theta &= \int_{\pi/2}^{\omega} \omega \, d\omega \\
-24.15 \left[ \left( \frac{\pi}{4} \right)^2 - \left( \frac{\pi}{2} \right)^2 \right] &= \frac{1}{2} \omega^2 \\
\omega &= 9.45 \text{ rad/sec}
\end{align*}
\]
Problem 17.73 (Page 418)

The disk shown has a mass of 20 kg and is originally spinning at the end of a strut with an angular velocity of $\omega = 60$ rad/sec. If it is then placed against the wall that has a coefficient of kinetic friction $\mu_k = 0.3$, determine a) the time required for the motion to stop, and b) the force in the strut BC during this time.

$$\sum_{\alpha} \dot{N}_b = 0.3N(0.15) = I_b \alpha$$

$$I_b = \frac{1}{2} (20)(0.15)^2 = 0.225 \text{ kg m}^2$$

$$\alpha = 0.2N$$

$$\sum F_x = -N + B \sin 30^\circ = 0$$

$$\sum F_y = B \cos 30^\circ - mg + 0.3N = 0$$

$$N = 0.5B = -2.887B + 654$$

$$B = 193.09 \text{ N}$$

$$N = 96.54 \text{ N}$$

$$\alpha = -0.2 \left( \frac{96.54}{96.54} \right) = 19.31 \text{ rad/s}^2$$

$$\omega = \omega_0 + \alpha t$$

$$0 = 60 - 19.31 \cdot t$$

$$t = 3.11 \text{ sec}$$
Problem 17.86 (Page 421)

The drum shown below has a weight of 50 lbs and has a radius of gyration of $k_A = 0.4$ ft. A 35 foot chain having a weight of 2 lb/ft is wrapped around the outer surface of the drum so that a chain length of $s = 3$ feet is suspended as shown. If the drum is originally at rest, determine its angular velocity after the end B of the chain has descended to $s = 13$ feet. Neglect the thickness of the chain.

\[
I_{A,\text{drum}} = \left( \frac{50}{32.2} \right) (0.4)^2 = 0.249 \text{ slug ft}^2
\]

\[
I_{A,\text{chain}} = \frac{2(35 - 5)}{32.2} (0.6)^2 = 0.0224(35 - 5)
\]

\[
\sum F_y = 2S = \frac{2S}{32.2} (0.6) \alpha
\]

\[
\left( \sum M_A = 0.6(2S) - \frac{2S}{32.2} (0.6) \alpha \right) = I_{A,\text{drum}} \alpha + I_{A,\text{chain}} \alpha
\]

\[
\alpha = 1.164 s = \omega \frac{d\omega}{d\theta}
\]

\[
\int_{3}^{13} 1.164 s \left( \frac{ds}{0.6} \right) = \int \omega \, d\omega
\]

\[
1.9398 \left[ \frac{13^2}{2} - \frac{3^2}{2} \right] = \frac{1}{2} \omega^2
\]

\[
\omega = 17.6 \text{ rad/s}
\]
Problem 17.90 (Page 429)

A rocket weighs 20,000 lbs and has a radius of gyration about the mass center G of \( k_G = 21 \) ft. when it is fired. Each of its engines has a thrust of \( T = 50,000 \) lbs. At a given instant in time, engine A suddenly fails to operate. Determine the angular acceleration \( \alpha \) of the rocket and the linear acceleration \( a_B \) of its nose B at this instant.

\[
\sum M_G = 1.5 \times 50,000 = (21)^2 \left( \frac{20,000}{32.174} \right) \alpha
\]

\[
\alpha = 0.2736 \text{ rad/sec}^2
\]

\[
\bar{a}_B = \bar{a}_G + \bar{a}_{B/c}
\]

\[
\sum F_y = 50,000 - 20,000 = \frac{20,000 a_c}{32.174}
\]

\[
a_c = 48.26 \text{ ft/sec}^2
\]

\[
\bar{a}_c = -0.2736(30) \hat{i} + 48.26 \hat{j}
\]

\[
\bar{a}_c = -8.208 \hat{i} + 48.26 \hat{j}
\]

\[
a_B = 48.95 \hat{r}_{80.3}^\prime
\]
Problem 17.96 (Page 430)

The spool shown has a mass of 100 kg and a radius of gyration of \( k_G = 0.3 \). If the coefficient of static and kinetic friction at A are \( \mu_s = 0.2 \) and \( \mu_k = 0.15 \), respectively, determine the angular acceleration of the spool if \( P \) is vertically up at 50 N.

\[
\sum F_x = F_A = 100 \, Q_G
\]
\[
\sum F_y = N_A + 50 - 100(9.81) = 0
\]
\[
\sum M_G = 50(0.25) - 0.4 F_A = 100(0.3)^2 \alpha
\]

For no slipping, \( Q_G = 0.4 \alpha \)
\( \alpha = 0.5 \, \text{r/s}^2 \)
\( Q_G = 0.4(0.5) = 0.2 \, \text{m/s}^2 \)
\( N_A = 931 \, \text{N} \)
\( F_A = 20 \, \text{N} \)
\( F_{A_{\text{max}}} = 0.2(931) = 186.2 \, \text{N} > 20 \, \text{N} \) (\( \therefore \) no slipping).
Problem 17.102 (Page 431)

The lawn roller shown has a mass of 80 kg and a radius of gyration \( k_g = 0.175 \text{ m} \). If the roller is pushed forward with a force 200 N when the handle is at 45°, determine its angular acceleration. The coefficient of static and kinetic friction between the ground and the roller are \( \mu_s = 0.12 \) and \( \mu_k = 0.10 \).

\[
I_g = (0.175^2)(80) = 2.45 \text{ kg m}^2
\]

\[
\sum M_G = F_x(2) = 2.45 \alpha
\]

\[
\sum F_y = N - 80(9.81) - (0.707)200 = 926.2 \text{ N}
\]

\[
N = 926.2 \text{ N}
\]

\[
\sum F_x = F_f - 200(0.707) = -80(2) \alpha
\]

\[
F_f = 12.25 \alpha = 141.4 - 16 \alpha
\]

\[
\alpha = 5.005 \text{ rad/s}^2
\]

\[
F_f = 61.31 \text{ N}
\]

\[
F_{w_k} = 0.12(926.2) = 111.14 \text{ N}
\]

:: roller rolls w/o slipping
The 16-lb bowling ball is cast horizontally onto a lane such that initially \( \omega = 0 \) and its mass center has a velocity \( v = 8 \) ft/sec. If the coefficient of kinetic friction between the lane and the ball is \( \mu_k = 0.12 \), determine the distance the ball travels before it rolls without slipping. For the calculations, neglect the finger holes in the ball and assume the ball has a uniform density.

\[
\sum F_x = 0.12N_A = \frac{16}{32.2} a
\]

\[
\sum F_y = N_A - 16 = 0
\]

\[
\sum M_c = 0.12N_A (0.375) = \frac{2}{5} \left( \frac{16}{32.2} \right)^2 (0.375) \alpha t^2
\]

\[N_A = 16 \text{ lb}\]

\[a = 3.864 \text{ ft/s}^2\]

\[\alpha = 25.76 \text{ rad/s}^2\]

When ball rolls w/o slipping \( v = 0.375 \omega \)

\[\omega = \omega_0 + \alpha t = \frac{V}{0.375} = 0 + 25.76 t\]

\[V = 9.66 \text{ ft} = V_0 + Qt = 8 - 3.864t\]

\[t = 0.592 \text{ sec}\]

\[S = S_0 + V_0t + \frac{1}{2} Qt^2 = 0 + 8(0.592) - \frac{1}{2}(3.864)(0.592)^2 = 4.06 \text{ ft}\]
Problem 17.108 (Page 433)

A 10-lb thin ring is given an initial angular velocity of 6 rad/sec when it is placed on a surface. If the coefficient of kinetic friction between the ring and the surface is $\mu_k = 0.3$, determine the distance the ring moves before it stops slipping.

$$\sum F_y = N - 10 = 0 \quad N = 10 \text{ lb}$$

$$\sum F_x = -0.3(10) = \frac{10}{32.174} a_c$$

$$a_c = 9.66 \text{ ft/s}^2$$

$$\sum M_o = I_o \alpha = 0.3(10)(\frac{6}{12})$$

$$= \frac{10}{32.174} \left(\frac{6}{12}\right)^2$$

$$\alpha = -19.32 \text{ rad/s}^2 = \frac{dw}{dt}$$

$$\omega = 0 + \alpha t$$

$$= 6 - 19.32 t$$

$$38.64 t = 6$$

$$t = 0.155 \text{ sec}$$

$$V_c = \frac{1.5 \text{ ft/s}}{\frac{ds}{dt}}$$

$$S = S_o + V_{c0} t + \frac{1}{2} a_c t^2$$

$$= 0 + 0 + \frac{1}{2}(9.66)(0.155)^2$$

$$= 0.116 \text{ ft} = 1.4 \text{ in}$$