Problem 12.11 (Page 15)

The acceleration of a particle as it moves along a straight line is given by $a = 2t - 1 \text{ (m/s}^2\text{)}$, where $t$ is in seconds. If $s = 1 \text{ m}$ and $v = 2 \text{ m/sec}$ when $t = 0$, determine a) the particle’s velocity and position when $t = 6$ seconds, and b) the total distance the particle travels during this time period.

\[
a = \frac{dv}{dt} = 2t - 1
\]

\[
\int dv = \int (2t - 1) \, dt
\]

\[
v = 2t^2 - t = \frac{ds}{dt}
\]

\[
\int ds = \int (2t^2 - t) \, dt
\]

\[
s = 1 + 2t + \frac{t^3}{3} - \frac{t^2}{2}
\]

When $t = 6$ sec

\[
v = 32 \text{ m/s}
\]

\[
s = 67 \text{ m}
\]

Since $v \neq 0$, $d = 67 - 1 = 66 \text{ m}$
Problem 12.26 (Page 17)

Ball A is released from rest at a height of 40 ft. at the same time that a second ball B is thrown upward 5 ft. from the ground. If the balls pass one another at a height of 20 ft., determine the speed at which ball B was thrown upward.

For ball A:

\[ a_y = -g = \frac{dv_y}{dt} \]

\[ \int_{0}^{t} (-g) \, dt = \int_{0}^{t} dv_y \]

\[ v_y = \frac{dy}{dt} = -gt \]

\[ \int_{40}^{20} dy = -g \int_{0}^{t} dt = -20 = -\frac{1}{2} gt^2 \]

\[ t = 1.115 \text{ sec} \]

For ball B:

\[ a_y = -g = \frac{dv_y}{dt} \]

\[ v_y = \frac{dy}{dt} = V_0 - gt \]

\[ \int_{0}^{t} (V_0 - gt) \, dt = \int_{0}^{20} dy \]

\[ V_0 t - \frac{1}{2} gt^2 = 15 = V_0 (1.115) + \frac{1}{2} (32.17)(1.115)^2 \]

\[ V_0 = 31.4 \text{ ft/s} \uparrow \]
Problem 12.43 (Page 25)

The a-s graph for a jeep traveling along a straight road is given for the first 300 m of its motion. At $s = 0$, $v = 0$. Construct the v-s graph.

\[
\frac{a}{s} = \frac{2}{200} \quad a = 0.01s \text{ m/s}^2
\]

$200 < s \leq 300$ m

\[
\frac{a}{300-s} = \frac{2}{300-200}
\]

$a = -0.02s + 6 \text{ m/s}^2$

\[
a = \frac{dv}{ds}
\]

\[
\int v \, dv = \int 0.01s \, ds
\]

$v = 0.1s \text{ m/s}$

At $s = 200$ m, $v = 20 \text{ m/s}$

$200 < s \leq 300$

\[
\int_{20}^{s} v \, dv = \int_{20}^{s} (-0.02s + 6) \, ds
\]

$v = \sqrt{-0.02s^2 + 12s - 1200} \text{ m/s}$

At $s = 300$ m, $v = 24.5 \text{ m/s}$
Problem 12.46 (Page 26)

A race car starting from rest travels along a straight road. For 10 seconds it has an acceleration according to the graph as shown.

a) Construct the v-t graph that describes the motion, and
b) Determine the distance traveled in 10 seconds.

\[ a = \frac{dv}{dt} \]
\[ \int dv = \int \frac{1}{6} t^2 \, dt \]
\[ v = \frac{1}{18} t^3 \]

For \( t = 6 \) s \( v = 12 \) m/s
\[ \int_{12}^{v} dv = \int_{6}^{t} 6 \, dt \]
\[ v = 6t - 24 \text{ m/s} \]

For \( t = 10 \) sec \( v = 36 \) m/s
\[ v = \frac{dv}{dt} \]
\[ \int_{0}^{s} ds = \int_{0}^{t} \frac{1}{18} t^3 \, dt \]
\[ s = \frac{1}{72} t^4 \]

For \( t = 6 \) \( s = 18 \) m

\[ \int_{18}^{s} ds = \int_{6}^{t} (6t - 24) \, dt \]
\[ s = 3t^2 - 24t + 54 \]

For \( t = 10 \) sec \( s = 114 \) m
A ball is thrown from a tower with a velocity of 20 ft/sec as shown. Determine a) the x and y coordinates to where the ball strikes the slope, and b) the speed at which the ball hits the ground.

\[
v_x = 20 \left( \frac{3}{s} \right) = 12 \text{ ft/s} = \frac{dx}{dt}
\]

\[
\int_{0}^{t} 12 \, dt = \int_{0}^{x} dx
\]

\[
12t = x = 20 + d
\]

\[
a_y = -q = \frac{dv_y}{dt}
\]

\[
-\int_{0}^{t} q \, dt = \int_{0}^{v_y} dv_y
\]

\[
v_y = \frac{dy}{dt} = 20 \left( \frac{v}{s} \right) - q \, t = \frac{dy}{dt}
\]

\[
\int_{0}^{t} (16 - q \, t) \, dt = \int_{0}^{dy} dy
\]

\[
16t - \frac{32.17}{2} t^2 = .5d - 80
\]

or

\[
16t^2 - 10t - 90 = 0
\]

\[
t = 2.70 \text{ sec}
\]

\[
X = 12 \times (2.70) = 32.4 \text{ ft}
\]

\[
y = .5d = .5(12.4) = 6.2 \text{ ft}
\]

\[
V_x = \text{Constant} = 12 \text{ ft/s} \quad V_y = 16 - 32.17(2.7) = -70.86
\]

\[
V = \sqrt{(12)^2 + (-70.86)^2} = 71.87 \text{ ft/s}
\]
Problem 12-111 (Page 57)

At a given instant in time a train engine at \( E \) has a speed of 20 m/s and a total acceleration of 14 m/s\(^2\) acting in the direction shown. Determine the rate of increase in the train's speed along the track and the radius of curvature \( \rho \) of its path at this instant in time.

\[
\alpha = 14.0 \text{ m/s}^2
\]

\[
\alpha_t = 14 \cos 75^\circ
\]

\[
= 3.62 \text{ m/s}\!
\]

\[
\alpha_n = 14 \sin 75^\circ
\]

\[
= 13.52 \text{ m/s}\!
\]

\[
= \frac{v^2}{\rho} = \frac{(20)^2}{\rho}
\]

\[
\rho = 29.58 \text{ m}
\]