The path of a particle is defined by \( y^2 = 4kx \), and the component of the velocity along the \( y \)-axis is \( v_y = ct \), where both \( k \) and \( c \) are constants. Determine the \( x \) and \( y \) components of acceleration.

\[
y^2 = 4kx
\]

\[
z \frac{dy}{dt} = 4k \frac{dx}{dt} \quad \text{or} \quad 2y v_y = 4k v_x
\]

\[
z \left( \frac{dy}{dt} \right)^2 + 2y \frac{d^2y}{dt^2} = 4k \frac{d^2x}{dt^2}
\]

\[
z v_y^2 + 2y a_y = 4k a_x
\]

\[
v_y = ct \quad \quad \frac{dv_y}{dt} = c = a_y
\]

\[
z (ct)^2 + 2yc = 4ka_x
\]

\[
a_x = \frac{c}{2k} (y + ct^2)
\]
Problem 12.79 (Page 44)

When a rocket reaches an altitude of 40 meters it begins to travel along a parabolic path \((y - 40)^2 = 160 \, x\), where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at \(v_y = 180 \, m/s\), determine the magnitude of the rocket's velocity and acceleration when it reaches an altitude of 80 meters.

\[
(y - 40)^2 = 160 \, x \\
2(y - 40) \frac{dy}{dt} = 160 \frac{dx}{dt} \\
2(80 - 40)(180) = 160 \, V_x \\
V_x = 90 \, m/s
\]

\[
V = \sqrt{V_y^2 + V_x^2} = \sqrt{(180)^2 + (90)^2} = 201.25 \, m/sec
\]

\[
2(\frac{dy}{dt}) \frac{dy}{dt} = 160 \frac{dV_x}{dt} \\
2(180)^2 = 160 \, a_x \\
a_x = 405 \, m/s^2
\]

\[
sin\alpha: \frac{d^2 y}{dt^2} = 0
\]

\[
a_x \neq 0
\]
Problem 12.85 (Page 45)

It was observed that a football player kicked a football a horizontal distance of 126 feet during a time of 3.6 seconds. Determine the initial speed of the ball and the angle at which it was kicked.

\[ V_x = V_0 \cos \theta = \text{constant} t \]

\[ V_x = \frac{dx}{dt} \]

\[ \int_{0}^{126} dx = \int_{0}^{t} V_0 \cos \theta \, dt \]

\[ 126 = V_0 \cos \theta (3.6) \]

\[ V_0 \cos \theta = 35 \]

\[ g_y = -32.2 \frac{ft}{s^2} = \frac{dV_y}{dt} \]

\[ V_y = V_0 \sin \theta - 32.2t = \frac{dy}{dt} \]

\[ y_f = V_0 \sin \theta t - 16.1t^2 \]

When \( x = 126 \quad y_f = 0 \)

\[ 0 = V_0 \sin \theta t - 16.1t^2 \]

\[ V_0 \sin \theta = 57.96 \]

\[ V_0 \frac{\sin \theta}{\cos \theta} = \frac{57.96}{35} \]

\[ \theta = \tan^{-1} 1.656 \]

\[ \theta = 58.87^\circ \]

\[ V_0 = 67.71 \frac{ft}{s} \]