Problem 12.79 (Page 44)

When a rocket reaches an altitude of 40 meters it begins to travel along a parabolic path \((y - 40)^2 = 160 \, x\), where the coordinates are measured in meters. If the component of velocity in the vertical direction is constant at \(v_y = 180 \, \text{m/s}\), determine the magnitude of the rockets velocity and acceleration when it reaches an altitude of 80 meters.

\[
(y - 40)^2 = 160 \, x
\]

\[
2 (y - 40) \frac{dy}{dt} = 160 \frac{dx}{dt}
\]

\[
2 (80 - 40)(180) = 160 \, V_x
\]

\[
V_x = 90 \, \text{m/s}
\]

\[
V = \sqrt{V_y^2 + V_x^2}
\]

\[
V = \sqrt{(180)^2 + (90)^2} = 201.25 \, \text{m/sec}
\]

\[
2 (\frac{dy}{dt}) \frac{dy}{dt} = 160 \frac{dV_x}{dt}
\]

\[
2(180)^2 = 160 \, a_x
\]

\[
a_x = 405 \, \text{m/s}^2
\]

\(a_y = 0\)
A fireman standing on a ladder as shown directs the flow of water from his hose to the fire at B. Determine the velocity of the water at A if it is observed that the hose is held at $\theta = 20^\circ$.

\[ V_{x_0} = V_A \cos 20^\circ = \text{constant} \]

\[ \dot{x} = \frac{dx}{dt} \]

\[ x = V_A \cos 20^\circ t = 60 \]

\[ V_A t = 63.85 \]

In the $y$-direction:

\[ q = \frac{dV_y}{dt} \]

\[ \int dV_y = \int_0^t -q \, dt \]

\[ V_y = V_{y_0} - q \frac{t}{2} = -V_A \sin 20^\circ - qt \]

\[ \frac{dy}{dt} = \frac{dV_y}{dt} \]

\[ y = -30 \text{ ft} = -V_A \sin 20^\circ t - \frac{1}{2} qt^2 \]

\[ 0.342 V_A t + 16.085 t^2 = 30 \quad V_A t = 63.85 \]

\[ t = 0.7124 \text{ sec} \]

\[ V_A = 89.63 \text{ ft/sec} \]
Problem 12.92 (Page 47)

A man stands 60 feet from a wall and throws a ball with an initial speed of \( v_0 = 50 \text{ ft/sec} \). Determine the angle \( \theta \) at which he must release the ball in order for it to strike the wall at the highest point possible without hitting the ceiling. The room has a ceiling height of 20 feet as shown. Determine the maximum height \( h \) of the ball as it hits the wall.

\[
V_x = V_0 \cos \theta = \text{constant} = \frac{dx}{dt}
\]

\[
\int V_0 \cos \theta \, dt = \int dx
\]

\[
V_0 \cos \theta \, t = 60
\]

In the \( y \) direction,

\[
-q = \frac{dV_y}{dt} = \int_{-q}^{t} q \, dt = \int dV_y = V_y - V_0 \sin \theta
\]

\[
V_y = 0 \text{ when } y = y_{\text{max}} , \text{ Let } t = t' = \text{time to reach } y_{\text{max}}
\]

\[
0 = V_0 \sin \theta - qt' \quad \text{or} \quad t' = \frac{V_0 \sin \theta}{q}
\]

\[
\int_{0}^{t} (V_0 \sin \theta - qt) \, dt = \int_{0}^{y} dy = y
\]

\[
y = V_0 \sin \theta \, t - \frac{1}{2} q \, t^2
\]

At \( y = y_{\text{max}} = 15 \),

\[
15 = \frac{V_0^2 \sin^2 \theta}{2q} \quad \text{or} \quad \theta = 38.41^\circ
\]
Problem 12.92 (Continued)

A man stands 60 feet from a wall and throws a ball with an initial speed of $v_o = 50 \text{ ft/sec}$. Determine the angle $\theta$ at which he must release the ball in order for it to strike the wall at the highest point possible without hitting the ceiling. The room has a ceiling height of 20 feet as shown. Determine the maximum height $h$ of the ball as it hits the wall.

From $V_x = V_o \cos \theta = \text{constant} = \frac{dx}{dt}$

When $V_o = 50 \text{ ft/sec}$

\[ 50 \cos (38.43^\circ) t = 60 \]

\[ t = 1.532 \text{ seconds, time for ball to hit the wall} \]

From the equation for the vertical position

\[ y = V_y \sin \theta t - \frac{1}{2} gt^2 \]

\[ h = 50 \sin 38.43^\circ (1.532) - \frac{1}{2} (32.17)(1.532)^2 \]

\[ = 14.8 \text{ ft} \]
Problem 12-115 (Page 58 - 11th)

A truck travels in a circular path as shown having a radius of 50 meters at a speed of 4 m/sec. At \( s = 0 \), its speed increases by \( \dot{v} = 0.05 \) m/sec\(^2\), where \( s \) is in meters. Determine its velocity and acceleration when it has moved \( s = 10 \) meters.

\[
\frac{dv}{dt} = 0.05 \quad s = v \frac{dv}{ds}
\]

\[
\int_0^{10} 0.05 \ s \ ds = \int_4^v v \ dv
\]

\[
0.05 \ s^2 \bigg|_0^{10} = \frac{v^2}{2} \bigg|_4
\]

\[
v_t = 4.58 \ m/s \quad \text{tangential velocity}
\]

\[
a_t = 0.05(10) = 0.5 \ m/sec^2
\]

\[
a_n = \frac{v_t^2}{r} = \frac{(4.58)^2}{50} = 0.4195 \ m/sec^2
\]

\[
a = \sqrt{a_t^2 + a_n^2} = 0.653 \ m/sec^2
\]
Problem 12.122 (Page 60)

A ball is ejected horizontally from a tube with a speed of 8 m/sec as shown. Find a) the equation of the path, \( y = f(x) \), and b) the ball’s velocity, plus the normal and tangential components of acceleration when \( t = 0.25 \) sec.

\[
\begin{align*}
V_x &= 8 \text{ m/s} = \text{constant} = \frac{dx}{dt} \\
X &= 8t \\
\frac{dy}{dt} &= -g \\
\int_{y_0}^{y} dy &= -\int_{0}^{t} gt \, dt \\
y &= -\frac{1}{2}gt^2 \\
y &= -\frac{1}{2} g \left( \frac{x}{8} \right)^2 \\
y &= -0.0766 x^2 \quad \text{(path)}
\end{align*}
\]

For \( t = 0.25 \) sec
\[
\begin{align*}
V_x &= 8 \text{ m/s} \\
V_y &= -2.4525 \text{ m/s} \\
V_t &= \sqrt{8^2 + (-2.4525)^2} = 8.367 \text{ m/s} \\
\theta &= \tan^{-1} \frac{2.4525}{8} = 17.04^\circ \\
Q_x &= 0 \\
Q_y &= -9.81 \text{ m/s} \\
Q_n &= 9.81 \cos 17.04^\circ = 9.38 \text{ m/s}^2 \\
Q_t &= 9.81 \sin 17.04^\circ = 2.88 \text{ m/s}^2
\end{align*}
\]