MAE 3360 ENGINEERING ANALYSIS

Summer 2009

DEPARTMENT OF MECHANICAL
AND
AEROSPACE ENGINEERING

UNIVERSITY OF TEXAS AT ARLINGTON

Exam #1

CLOSED BOOKS and NO CALCULATORS

July 1, 2009
Time Limit: 1 hr 30 min

(This exam has 9 pages)

LAST NAME: ______________

FIRST NAME: ______________________

92/94

Highest score
1. (20 pts) TRUE OR FALSE  a correct answer scores 2 points, there is no penalty for incorrect response. Note that if a statement would only be true under certain conditions (conditionally true), then it is false.

*Write "T" for true statement and "F" for false statement.

\[ \begin{align*}
\text{T} & \quad \frac{d^2y}{dx^2} + y^2 = 2 \text{ is a linear ODE.} \\
\text{F} & \quad x^2y'' + 3xy' + 5y = \cos x \text{ is a nonlinear ODE.} \\
\text{F} & \quad \frac{d^2y}{dx^2} + \sin x \frac{dy}{dx} + y = 2x^3 \text{ is a nonlinear ODE.} \\
\text{T} & \quad y' + \frac{x}{y} = 0 \text{ is a linear ODE.} \\
\text{F} & \quad \text{The nonhomogeneous solution of } x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = 1 \text{ can be obtained by the method of undetermined coefficient.} \\
\text{F} & \quad \text{The nonhomogeneous solution of } \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2x^{3/2} \text{ can be obtained by the method of undetermined coefficient.} \\
\text{F} & \quad \text{The nonhomogeneous solution of } \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 2x^{3/2} \text{ can be obtained by the method of variation of parameters.} \\
\text{F} & \quad \text{The nonhomogeneous solution of } x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = 2x^{3/2} \text{ can be obtained by the method of variation of parameters.} \\
\text{F} & \quad x^2y'' + 3y = 0 \text{ is an equidimensional ODE.} \\
\text{F} & \quad \text{The set of functions } \{x, x^2\} \text{ is linearly independent.}
\end{align*} \]
2a. (10 pts) Determine the solution to: \( x \frac{dy}{dx} + y = e^x; \ y(1) = 2 \)

\[
\frac{dy}{dx} + \frac{1}{x} y = \frac{1}{x} e^x
\]

\[
\Rightarrow \text{ Integrating factor } = e^\int \frac{1}{x} \, dx = e^\ln x = x
\]

\[
\Rightarrow x \left[ \frac{dy}{dx} + \frac{1}{x} y \right] = e^x
\]

\[
\Rightarrow \frac{d}{dx}(xy) = e^x
\]

\[
\Rightarrow xy = \int e^x \, dx + C = e^x + C
\]

\[
\Rightarrow y = \frac{e^x + C}{x}
\]

I.C. \( y(1) = 2 \Rightarrow 2 = e + C \)

\[
\Rightarrow C = 2 - e
\]

\[
\boxed{y = \frac{e^x + 2 - e}{x}}
\]
2b. (10 pts) Determine the general solution to: \((x + y)^2 \, dx + (2xy + x^2 - 1) \, dy = 0\)

\[
\frac{\partial M}{\partial y} = 2x + 2y \quad \Rightarrow \quad \text{Exact Differential}
\]
\[
\frac{\partial N}{\partial x} = 2y + 2x \quad \Rightarrow \quad M = \frac{\partial \phi}{\partial x} \quad \text{and} \quad N = \frac{\partial \phi}{\partial y}
\]
\[
\phi = C \quad \text{is the solution}
\]

(i) \[
\frac{\partial \phi}{\partial x} = M = x^2 + xy + y^2
\]
\[
\Rightarrow \quad \phi = \frac{x^3}{3} + x^2y + x^2 + f_1(y)
\]

(ii) \[
\frac{\partial \phi}{\partial y} = N = 2xy + x^2 - 1
\]
\[
\Rightarrow \quad \phi = xy^2 + xy - y + f_2(x)
\]

Comparing (i) and (ii) \(\Rightarrow\) \(f_1(y) = -y\) \(\Rightarrow\) \(f_2(x) = \frac{x^3}{3}\)
\[
\phi = \frac{x^3}{3} + x^2y + xy^2 - y
\]
\[
\therefore \quad \frac{x^3}{3} + x^2y + xy^2 - y = C
\]
2c. (10 pts) Determine the general solution to: \((xy + y^2)dx - x^2dy = 0\)

Homogeneous to the order 2.

Let \(y = ux \Rightarrow dy = udx + xdu\)

D.E. becomes:

\[
(x(u) + (ux)^2)dx - x^2(udx + xdu) = 0
\]

\[
\Rightarrow (ux^2 + u^2x^2)dx - x^2udx - x^3du = 0
\]

\[
\Rightarrow u^2x^2dx = x^3du
\]

\[
\Rightarrow \frac{dx}{x} = \frac{du}{u^2}
\]

\[
\Rightarrow \ln x = -\frac{1}{u} + C
\]

\[
\Rightarrow (\ln x = -\frac{x}{y} + C)
\]
3. (15 pts) Determine the general solution to:

(a) \( \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0 \)

\[
\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0 \\
y = e^{mx} \\
\Rightarrow m^2 + m = 0 \\
\Rightarrow m = 0 \quad \text{or} \quad m = -\frac{1}{2}
\]

\[
y = c_1 + c_2 e^{-\frac{1}{2}x}
\]

(b) \( \frac{d^2y}{dx^2} + 8 \frac{dy}{dx} + 16y = 0 \)

\[
y = e^{mx} \\
\Rightarrow m^2 + 8m + 16 = 0 \\
\Rightarrow (m + 4)^2 = 0 \\
\Rightarrow m = -4 \quad \text{a double root}
\]

\[
y = (c_1 + c_2 x) e^{-4x}
\]

(c) \( \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 5y = 0 \)

\[
y = e^{mx} \\
\Rightarrow m^2 - 4m + 5 = 0 \\
\Rightarrow m = \frac{4 \pm \sqrt{4^2 - 4 \cdot 5}}{2} \\
\Rightarrow m = 2 \pm i
\]

\[
y = e^{2x} (c_1 \cos x + (2 \sin x))
\]
4. (20 pts) Determine the homogeneous solutions and write down the appropriate trial form of the non-homogeneous (particular) solution for the following problems. You do not have to solve for the coefficients in your proposed non-homogenous solution.

(a) \[ \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = e^{2x}(\cos x - 3 \sin x) \]

\[ y_h = m^2 - 2m + 2 = 0 \implies m = \frac{2 \pm \sqrt{2^2 - 4 \cdot 1}}{2} = 1 \pm i \]

\[ \implies y_h = e^x (C_1 \cos x + C_2 \sin x) \]

\[ y_p = e^{2x} (A \cos x + B \sin x) \]

(b) \[ \frac{d^2y}{dx^2} + 4y = 3 \sin 2x \]

\[ y_h = m^2 + 4 = 0 \implies m = \pm 2i \]

\[ \implies y_h = C_1 \cos 2x + C_2 \sin 2x \]

\[ y_p = A \cos 2x + B \sin 2x \]
(c) \[ \frac{d^2 y}{dx^2} \frac{dy}{dx} = -3 \]

\[ y_h : \quad h^2 - m = 0 \quad \Rightarrow \quad h = 0, \quad 1 \quad \Rightarrow \quad y_h = C_1 + C_2 e^x \]

Modified as needed

\[ y_p = A(x) \]

(d) \[ \frac{d^2 y}{dx^2} = -3x^2 + 2x - 1 \]

\[ y_h : \quad h^2 = 0 \quad \Rightarrow \quad m = 0 \quad \Rightarrow \quad y_h = A + Bx \]

\[ y_p = A x^4 + B x^3 + C x^2 \]

\[ \text{Modified as needed} \quad (\text{Handwritten}) \]
5. (15 pts) Solve \( \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} - 2y = 4x^2; y(0) = 1, \frac{dy}{dx}(0) = 4 \)

\[ y_1 = e^{-x}, \quad y_2 = e^{2x} \]

\[ y_p = A_2 x^2 + A_1 x + A_0 \]

\[ y_p' = 2A_2 x + A_1 \]

\[ y_p'' = 2A_2 \]

\[ 2A_2 - (2A_2 x + A_1) - 2(A_2 x^2 + A_1 x + A_0) = 4x^2 \]

\[ (-2A_2) x^2 + (-2A_1 - 2A_0) x + (2A_2 - A_1 - 2A_0) = 4x^2 \]

\[ -2A_2 = 4 \quad \Rightarrow \quad A_2 = -2 \]

\[ -2A_1 - 2A_0 = 0 \quad \Rightarrow \quad A_1 = -A_2 = 2 \quad A_0 = (2A_2 - A_1)/2 = -3 \]

\[ y_p = -2x^2 + 2x - 3 \]

\[ y = y_h + y_p = C_1 e^{-x} + C_2 e^{2x} - 2x^2 + 2x - 3 \]

Apply 1. C.

(i) \( y(0) = C_1 + C_2 - 3 = 1 \quad \Rightarrow \quad C_1 + C_2 = 4 \quad \text{①} \)

(ii) \( y'(0) = -C_1 + 2C_2 + 2 = 4 \quad \Rightarrow \quad -C_1 + 2C_2 = 2 \quad \text{②} \)

① + ② \( \Rightarrow \quad C_1 = 2, \quad C_2 = 2 \)

\[ y = 2 e^{-x} + 2 e^{2x} - 2x^2 + 2x - 3 \]