* Solution of higher order linear (homogeneous) ODE with constant coefficients.

General: For a given ODE with constant coefficient, linear & homogeneous.

Main 3 possibilities according to the roots of characteristic equation.

(1) Two real & distinct roots: $r$.

The general solution is given by:

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$m_1, m_2$ are two real & distinct roots of quadratic equation obtained from given ODE.

(2) Complex roots: $r$.

From characteristic equation, if the roots are:

$$m_1, m_2 = a + bi$$

then the solution of ODE is given by:

$$y = e^{ax} (A_1 \cos bx + A_2 \sin bx)$$
(3) **Double Root**

If the solution of characteristic equations results in a double root, then the solution of the given ODE is given by,

\[ y = A_1 e^{wx} + xA_2 e^{wx} \]

**Examples:**

\[ y'' - y' - 30y = 0 \]

**Solution:** From the given ODE, the characteristic equation is given by...

\[ m^2 - m - 30 = 0 \]

\[ \Rightarrow (m-6)(m+5) = 0 \]

\[ \Rightarrow m = 6 \quad \text{or} \quad m = -5 \]

two real & distinct roots.

\[ \therefore y_1 = A_1 e^{6x} \quad y_2 = A_2 e^{-5x} \]

The general solution

\[ y = y_1 + y_2 \]

\[ y = A_1 e^{6x} + A_2 e^{-5x} \]
\[ x + 60x + 500x = 0. \]

For the given O.D.E, the characteristic equation,

\[ m^2 + 60m + 500 = 0 \]

\[ \Rightarrow m^2 + 50m + 10m + 500 = 0 \]

\[ \Rightarrow (m+50)(m+10) = 0 \]

\[ \Rightarrow m = -50, \text{ and } -10. \]

two distinct real roots.

\[ \Rightarrow \text{The solution is given by,} \]

\[ y_1 = A_1 e^{m_1 x} = A_1 e^{-50x} \]

\[ y_2 = A_2 e^{m_2 x} = A_2 e^{-10x} \]

\[ \Rightarrow \text{The general solution is,} \]

\[ y = y_1 + y_2 \]

\[ y = A_1 e^{-50x} + A_2 e^{-10x}. \]

\[ \text{Ex: } \]

\[ y'' + 7y' = 0. \]

\[ \Rightarrow \text{The characteristic equation is given by,} \]

\[ m^2 + 7m = 0 \]

\[ m(m + 7) = 0 \]

\[ \Rightarrow m = 0, \text{ or } m = -7 \]

two distinct real roots.

\[ \Rightarrow \text{The general solution is,} \]

\[ y = A_1 e^{0x} + A_2 e^{-7x} \]

\[ \Rightarrow \left| y = A_1 + A_2 e^{-7x} \right| \]