MAE 3360  HW #3
ENGINEERING ANALYSIS

Exercises 3.3 – HOMOGENEOUS LINEAR EQUATIONS
WITH CONSTANT COEFFICIENTS.

3. \( y'' - y' - 6y = 0 \)

The auxiliary equation is
\( m^2 - m - 6 = 0 \)

\( \therefore m = 3 \) and \( m = -2 \)

\( \therefore y = c_1 e^{3x} + c_2 e^{-2x} \)

5. \( y'' + 8y' + 16y = 0 \)

The auxiliary equation is
\( m^2 + 8m + 16 = 0 \)

\( \therefore m = -4 \) and \( m = -4 \)

\( \therefore y = c_1 e^{-4x} + c_2 xe^{-4x} \)

12. \( 2y'' + 2y' + y = 0 \)

The auxiliary equation is
\( 2m^2 + 2m + 1 = 0 \)

\( \therefore m = -\frac{1}{2} + \frac{i}{2} \)

\( \therefore y = e^{-\frac{x}{2}} [c_1 \cos \frac{x}{2} + c_2 \sin \frac{x}{2}] \)
Exercises 3.4 - Undetermined Coefficients

3. \( y'' - 10y' + 25y = 30x + 3 \)

The auxiliary equation is \( m^2 - 10m + 25 = 0 \)

\( \therefore m_1 = m_2 = 5 \)

\( y_c = c_1 e^{5x} + c_2 xe^{5x} \)

Assume \( y_p = Ax + B \)

\( \therefore y_p' = A \)

\( \therefore y_p'' = 0 \)

Substituting in the equation,

\( 0 - 10A + 25(Ax + B) = 30x + 3 \)

\( \therefore 25A = 30 \) and \( -10A + 25B = 3 \)

\( \therefore A = \frac{6}{5} \) and \( B = \frac{3}{5} \)

\( \therefore y_p = \frac{6}{5}x + \frac{3}{5} \)

\( \therefore y = c_1 e^{5x} + c_2 xe^{5x} + \frac{6}{5}x + \frac{3}{5} \)

10. \( y'' + 2y' = 2x + 5 - e^{-2x} \)

The auxiliary equation is \( m^2 + 2m = 0 \)

\( \therefore m_1 = -2 \) and \( m_2 = 0 \)

\( y_c = c_1 e^{-2x} + c_2 \)

Assume \( y_p = Ax^2 + Bx + Cx e^{-2x} \)

\( \therefore y_p' = 2Ax + B + C e^{-2x} + (-2)x C e^{-2x} \)

\( \therefore y_p'' = 2A + (-2)Ce^{-2x} - 2Ce^{-2x} + 4xCe^{-2x} \)
Substituting in the equation,
\[ 2A - 2Ce^{-2x} - 2Ce^{-2x} + 4Ax e^{-2x} + 4Ax + 2B + 2Ce^{-2x} \]
\[ -4Ce^{-2x} = 2x + 5 - e^{-2x} \]
\[ (2A + 2B) + 4Ax - 2Ce^{-2x} = 5 + 2x - e^{-2x} \]
\[ 4A = 2 \quad \therefore A = \frac{1}{2} \]

\[ 2A + 2B = 5 \quad \therefore B = 2 \]
\[ 2 - 2C = -1 \quad \therefore C = \frac{1}{2} \]

\[ y_p = \frac{1}{2} x^2 + 2x + \frac{1}{2} x e^{-2x} \]

\[ y = C_1 e^{-2x} + C_2 + \frac{1}{2} x^2 + 2x + \frac{1}{2} x e^{-2x} \]

13. \( y'' + 4y = 3\sin 2x \)

The auxiliary equation is
\[ m^2 + 4 = 0 \]
\[ m = 2i \quad \text{and} \quad m = -2i \]

\[ y_c = C_1 \cos 2x + C_2 \sin 2x \]

Assume \( y_p = Ax \cos 2x + Bx \sin 2x \)

\[ y_p = A \cos 2x - 2A \sin 2x + B \sin 2x + 2B \cos 2x \]

\[ y''_p = -2A \sin 2x - 2A \sin 2x - 4A \cos 2x + 2B \cos 2x \]
\[ + 2B \cos 2x - 4B \sin 2x \]

Substituting in the equation,
\(-2A \sin 2x - 4A \cos 2x + 4B \cos 2x - 4Bx \sin 2x \\
+ 4Ax \cos 2x + 4Bx \sin 2x = 3 \sin 2x\)

\[\therefore -4A \sin 2x + 4B \cos 2x = 3 \sin 2x\]

\[\therefore -4A = 3 \therefore A = -\frac{3}{4}\] and \(4B = 0 \therefore B = 0\]

\[\therefore y_p = -\frac{3}{4} x \cos 2x\]

\[y = c_1 \cos 2x + c_2 \sin 2x - \frac{3}{4} x \cos 2x\]

Exercises 3.6 - Cauchy Euler Equation

10. \(4x^2 y'' + 4x y' - y = 0\)

Consider \(y = x^m \therefore y' = mx^{m-1} \) and \(y'' = m(m-1)x^{m-2}\)

Substituting in the equation,

\[4x^2 [m(m-1)x^{m-2}] + 4x(mx^{m-1}) - y = 0\]

\[\therefore 4m(m-1) + 4m - 1 = 0\]

\[\therefore 4m^2 - 1 = 0\]

\[\therefore m = -\frac{1}{2} \text{ and } m = \frac{1}{2}\]

\[y = c_1 x^{\frac{1}{2}} + c_2 x^{-\frac{1}{2}}\]
11. \[ x^2 y'' + 5x y' + 4y = 0 \]

Consider \( y = x^m \), \( y' = mx^{m-1} \) & \( y'' = m(m-1)x^{m-2} \)

\[ x^2[m(m-1)x^{m-2}] + 5xmx^{m-1} + 4x^m = 0 \]

\[ m^2 + 4m + 4 = (m + 2)^2 = 0 \]

\[ m_1 = -2 \quad \text{and} \quad m_2 = -2 \]

\[ y = c_1x^{-2} + c_2x^{-2}\ln x \]