Problem 1)

\[ G(s) = \frac{3}{s^2 + 5s + 4} \]

\[ \lambda_1, \lambda_2 = -4, -1 \]

(i) Since we have 2 real poles, we cannot talk about damping and frequency. Time constants of the system are

\[ \tau_1 = 1 \quad \tau_2 = 0.25 \]

DC gain \( K_2 = 3/4 \).

(ii) Since system has 2 negative real poles, it is over damped.

(iii) 

\[ T_r = N/A \]

\[ \%\text{overshoot} = N/A \]
b) \[ G(s) = \frac{4}{s^2 + 2s + 2} \]

(i) \[ s_{1,2} = -1 \pm i \]

If we compare this system with a general second order system \( TF \),
\[ G(s) = \frac{k_2 \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

\[ \omega_n^2 = 2 \quad \omega_n = 1.41 \]
\[ 2\zeta \omega_n = 2 \quad \zeta = 0.707 \]
\[ k_2 \omega_n^2 = 4 \quad k_2 = 2 \]

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

(iii) since \( 0 < \zeta < 1 \), the system is underdamped.

\[ T_F = \frac{\pi - \arctan \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right)}{\sqrt{1 - \zeta^2} \omega_n} \]

\[ T_F = 2.362 \text{ sec} \]

\[ \% \text{ overshoot} = 100 \times \frac{\text{peak}}{\omega_n} = 4.3 \% \]
(c) \[ G(s) = \frac{3}{s^2 - 4s + 13} \]

(i) \[ s_{1,2} = 2 \pm 3i \]

Since the system has 2 complex poles with positive real parts, the system will have negative damping ratio.

\[
\begin{align*}
\omega_n &= 13 \\
\zeta &= -0.555 \\
\zeta &= 0.231 \\
\omega_d &= N/A
\end{align*}
\]

Since \( \zeta < 0 \), the system is unstable.

(ii) Since \( \zeta < 0 \), the system is unstable.

(iii) N/A since the system is unstable.
\[ G(s) = \frac{1}{s^2 + 4s + 13} \]

(i) \[ s_{1,2} = -2 \pm 3i \]

\[ \begin{align*}
\omega_n^2 &= 13 \\
2\zeta\omega_n &= 4 \\
K_2\omega_n^2 &= 1
\end{align*} \]

\[ \begin{align*}
\omega_n &= 3.607 \\
\zeta &= 0.555 \\
K_2 &= 0.0769
\end{align*} \]

\[ \omega_d = \omega_n \sqrt{1 - \zeta^2} \]

\[ \tau_d = \frac{\pi}{\omega_d} \]

(ii) Since \( 0 < \zeta < 1 \), the system is underdamped.

(iii) \[ \tau_r = \frac{\pi - \arctan \left( \frac{\sqrt{1 - \zeta^2}}{\zeta} \right)}{\sqrt{1 - \zeta^2} \omega_n} \Rightarrow \tau_r = 0.713 \text{ sec} \]

\[ \% \text{ overshoot} = 100 \times e^{-\frac{3\pi}{\sqrt{1 - \zeta^2}}} = 12.3 \% \]
Problem 2)

A first-order TF in terms of its DC gain and time constant can be written as: \[ \frac{k_1}{s+1} \]

\[ G_1(s) = \frac{2}{0.5s+1} = \frac{4}{s+2} \]

A second-order TF in terms of its DC gain, damping ratio and natural frequency: \[ \frac{k_2w_n^2}{s^2 + 2\zeta w_n s + w_n^2} \]

where \( k_2 > 0.1 \), \( \zeta \) and \( w_n \) are to be determined from the specs.

\[ \% \text{ overshoot} = e^{-\frac{7\pi}{\sqrt{1-\zeta^2}}} = 0.10 \]

\[ \Rightarrow \zeta = \frac{\ln\left(\frac{10}{10^{0.10}}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{10}{10^{0.10}}\right)}} = 0.59 \]

\[ \zeta = 0.59 \]
\[ T_s = \frac{[-\ln \frac{d}{100}]}{\frac{1}{\omega_n}} = \frac{\ln \frac{100}{d}}{\frac{1}{\omega_n}} \]

\[
\Rightarrow \omega_n = \frac{\ln \frac{100}{d}}{\frac{1}{T_s}} \]

where \( d = 2 \), \( T_s = 5 \)

\[ \omega_n = 1.326 \]

Using \( k_2, \gamma \) and \( \omega_n \), we can write

\[
G_2(s) = \frac{0.1758}{s^2 + 1.564s + 1.758} \]

Thus,

\[ G(s) = G_1(s) + G_2(s) \]

\[
G_1(s) = \frac{4s^2 + 6.4318s + 7.3836}{s^3 + 3.564s^2 + 1.986s + 3.516} \]
Problem 3)

\[ G(s) = \frac{10}{s^2 + 2s + 10} \]

i) Using Matlab, \( \omega_n = 3.1623 \)
\( \xi = 0.3162 \)

Poles:
\[ s_{1,2} = -1.0000 \pm 3.0000i \]

ii) \( \% \text{ overshoot}=100 \times e^{-\frac{\xi \omega_n}{\sqrt{1-\xi^2}}} = 35 \%
\)

5% settling time:
\[ T_s = \frac{3}{\xi \omega_n} = 3 \text{ sec} \]

\[ \tau_p = \frac{\pi}{\omega_n} \sqrt{1-\xi^2} = \frac{\pi}{3.1623 \sqrt{1-0.3162^2}} \approx 1.0417 \text{ sec} \]

\[ M_p = k_p \xi \omega_n e^{-\frac{\xi \omega_n}{\sqrt{1-\xi^2}}} \approx 1.351 \]
iv)
If we compare the step responses of the systems without a zero and with a zero, we can see that

- The response of the system with a zero starts initially in the wrong direction, that is, going away from the steady state value. This is due to "the non-minimum phase zero", that is, a zero with positive real part.
- Maximum overshoot and settling time of the system with the zero is also increased.
- A zero may also change the DC gain of a system.
MAE 4310 Design Project Assignment 3

1) \[ 0.1 \delta_a(t) + \delta_0(t) = V_{\text{serv}}(t) \]

\[ \Rightarrow (0.15 + 1) \delta_a(t) = V_{\text{serv}}(t) \]

\[ \Rightarrow \frac{\delta_a(t)}{V_{\text{serv}}(t)} = \frac{1}{0.15 + 1} \]

\[ \Rightarrow \text{Time constant } \tau = 0.1, \text{ DC gain } K = 1 \]

2) Let \[ T(s) = \frac{\delta_a(s)}{V_{\text{serv}}(s)} \]

The partial fraction expansion of \[ T(s) \] is

\[ T(s) = \frac{0.1738}{s + 1.2503} - \frac{0.0076}{s + 0.0582} + \frac{0.0638 s - 0.0035}{s^2 - 0.0025 s + 0.4746} \]

The second order TF corresponding to the complex poles is

\[ \frac{0.0638 s - 0.0035}{s^2 - 0.0025 s + 0.4746} \]

\[ \omega_n = 0.4746 \Rightarrow \omega_n = 0.6889 \]

\[ 2\zeta \omega_n = -0.0025 \Rightarrow \zeta = -0.0019 \]