Homework–5 (Due date: Thursday, Oct 19, 2006)

Textbook Readings:

1. Consider the satellite attitude control system below

\[
\begin{align*}
R(s) & \xrightarrow{K} K \xrightarrow{\frac{1}{Js^2}} C(s) \\
R(s) & \xrightarrow{\frac{K}{Js}} \xrightarrow{\frac{1}{s}} C(s) \quad K_h
\end{align*}
\]

Show that the output of this system exhibits continued oscillations by determining the damping ratio of the closed loop system. This system can be stabilized by the following control structure

\[
\begin{align*}
R(s) & \xrightarrow{K} K \xrightarrow{\frac{1}{Js^2}} C(s) \\
R(s) & \xrightarrow{\frac{K}{Js}} \xrightarrow{\frac{1}{s}} C(s) \quad K_h
\end{align*}
\]

If \( K/J = 4 \), what value of \( K_h \) will yield the damping ratio to be 0.6.

2. Determine the range of \( K \) for stability of a unity feedback control system whose open–loop transfer function is

\[
G(s) = \frac{K(s - 1)}{(s + 2)(s^2 + 2s + 2)}
\]

You should use Routh–Hurwitz Stability Criterion to answer this question.

3. Consider the unity–feedback control system with the following open–loop transfer function.

\[
G(s) = \frac{10}{s(s - 1)(s + 3)}
\]

Determine the stability of the closed loop system by using Routh–Hurwitz Method.

4. Consider the following characteristic polynomial

\[
s^3 + (4 + K)s^2 + 6s + 12
\]

For what values of \( K \) does this polynomial have roots with negative real parts? Use Routh–Hurwitz Method.
5. How many roots with positive real parts does each polynomial have.

(a) \( s^3 + s^2 - s + 1 \)
(b) \( s^4 + 2s^3 + 2s^2 + 2s + 1 \)
(c) \( s^3 + s^2 - 2 \)

Use Routh–Hurwitz Method to prove your answer.

6. MATLAB
Consider the following system

\[
\begin{array}{ccc}
  & 1 & \\
  & \frac{1}{\tau s + 1} & \\
  & & y  \\
\end{array}
\]

This system is often called "low pass filter". Suppose \( \tau = 2 \) and \( u(t) = 2 + \sin(wt) \) for \( w = 0.01, 0.1, 1, 10, 100 \). Use MATLAB to plot \( u(t) \) and \( y(t) \) on the same plot, using enough time to achieve steady–state. Based on these plots, explain the name "low pass filter".
1. In this section of the project, we use proportional feedback to stabilize the unstable aircraft dynamics and to obtain desirable flying qualities. This type of aircraft control system is known as a stability augmentation system (SAS). The output of the rate gyro will be used as negative feedback to the system and the sensitivity of the gyro will be adjusted to move the poles along the root-locus to desirable locations.

From our previous block diagrams, we note that the system from \( V_{\text{servo}} \) to bank angle \( \phi \) with \( V_{\text{gyro}} \) used for negative feedback is represented by

\[
G_a(s) = \frac{1}{0.1s + 1}, \quad G_p(s) = \frac{0.23s^3 + 0.079s^2 + 0.0789s + 0.0001}{s^4 + 1.286s^3 + 0.543s^2 + 0.6114s + 0.034}
\]

The value of the \( K_{\text{gyro}} \) given in Assignment 2 is 0.5. Our objective is to change this value to meet the specifications defined in Assignment 4 and characterized by the poles of \( G_{\text{des}}(s) \).

(i) Find the transfer function \( G_1(s) \) which represents to cascade connection of \( G_a(s) \) and \( G_p(s) \). You can use the `series` command in MATLAB to do this. The block diagram in Figure 1 can now be drawn as

\[
\begin{align*}
\begin{array}{c}
p_{\text{ref}} + v_{\text{gyro}} \\
G(s) \\
K_{\text{gyro}} \\
1/s
\end{array}
\end{align*}
\]

Figure 1: Block Diagram of Servo and Aircraft with Negative Feedback

where

\[
G_a(s) = \frac{1}{0.1s + 1}, \quad G_p(s) = \frac{0.23s^3 + 0.079s^2 + 0.0789s + 0.0001}{s^4 + 1.286s^3 + 0.543s^2 + 0.6114s + 0.034}
\]

(ii) Use MATLAB to plot the root-locus of \( G_1(s) \) as \( K_{\text{gyro}} \) is varied from 0 to \( \infty \).

(iii) Find the complex poles of \( G_{\text{des}}(s) \) obtained in Assignment 4. These poles correspond to other desired dutch roll dynamics.

(iv) Locate the points on the root-locus which are closest pole locations in (iii). Use the MATLAB command `rlocfind` to find the value of \( K_{\text{gyro}} \) that corresponds to these points and let \( K_{\text{gyro}} \) take on this value.
(v) Use the `feedback` command in MATLAB to find the closed-loop transfer function $T_1(s)$ from $p_{ref}$ to $p$ with $K_{gyro}$ set to the value obtained in (iv). The block diagrams in Figures 1, 2 can now be drawn as

![Block Diagram with Closed Loop](image)

Figure 3: Block Diagram with Closed Loop

(vi) Find the time constants, damping ratio and natural frequency of $T_1(s)$ using MATLAB, and see if they meet the specifications defined in Assignment 4. Note from the tables that slower spiral modes and faster roll modes are acceptable. Ignore the fastest pole in the system (the actuator pole) while performing this analysis.

2. SIMULINK

(i) Change the gain of the rate gyro in your model with the actual aircraft dynamics ("model1") from 0.5 V/deg to the value obtained in (iv) above. Next, make the feedback connection shown in Figure 1 by using the `sum` block from the "Linear" block library in SIMULINK. Double click on the block and set the signs field to "+-" for negative feedback. Use a step input as $p_{ref}$. Simulate the system and plot $p$, $\phi$ and $\delta_a$. Compare these outputs to the corresponding outputs from model2 with $G_{des}(s)$. 