Homework–2 (Due date: Thursday, Sep 21, 2006)

Textbook Readings: Sections 5–1, 5–2, 5–3, 5–4

1. Consider the system

\[ \dddot{y}(t) + 8\ddot{y}(t) + 25y(t) = 2\dot{u}(t) + u(t) \]

with \( y(0) = 1, \dot{y}(0) = 0 \) and \( \ddot{y}(0) = 0 \)

(i) Determine the transient and steady-state response to the inputs

(a) \( u(t) = 3 \times 1(t) \)
(b) \( u(t) = t^2 \)
(c) \( u(t) = \cos 3t \)

by using Laplace transforms and partial fraction expansions. You may use the table of Laplace transforms (pp. 17-18) of the text and MATLAB for the partial fraction expansions.

(iv) Determine whether \( \lim_{t \to \infty} y(t) \) exists for inputs a–c, and if so, use the Final Value Theorem to evaluate it.

2. Consider the system shown below

\[ \begin{align*}
\textbf{u} & \quad \textbf{G}(s) \\
& \quad \textbf{y}
\end{align*} \]

Use the Final Value Theorem to find \( \lim_{t \to \infty} y(t) \), if it exits, for the following cases:

(i) \( G(s) = \frac{1}{2s^2 + s + 4}, \quad U(s) = \frac{1}{s} \)
(ii) \( G(s) = \frac{1}{s^3 + 3s - 4}, \quad U(s) = 1 \)
(iii) \( G(s) = \frac{s^2 + 2}{4s^2 + 2}, \quad U(s) = \frac{1}{s^2} \)
(iv) \( G(s) = \frac{4}{s - 1}, \quad U(s) = \frac{1}{s^2} \)
(v) \( G(s) = \frac{1}{s + 2}, \quad U(s) = \frac{s}{s^2 + 4} \)

3. Find the DC gain and time–constant \( \tau \) of the following first-order systems:

(i) \( G(s) = \frac{9}{s + 3} \)
(ii) \( G(s) = \frac{1}{s + 0.5} \)
(iii) \( G(s) = \frac{14}{s + 1} \)
4. MATLAB

(i) Create pole-zero maps for the systems in Problem 3. To do this, type

```matlab
num = numerator of G(s), den = denominator of G(s);
>> sys1=tf(num,den)
>> pzmap(sys1)
>> axis([xmin xmax ymin ymax])
>> sgrid
```

The second command creates the `sys1` with transfer function \( \frac{num}{den} \). The `axis` command sets the minimum value of the x-axis of the plot to \( xmin \), the maximum value of the x-axis of the plot to \( xmax \). The corresponding values for the y-axis are \( ymin \) and \( ymax \). You can choose the numbers \( xmin, xmax, ymin, ymax \).

(ii) Plot the impulse and step responses of the systems in Problem 3. To do this type

```matlab
>> impulse(sys1)
>> step(sys1)
```

(iii) Describe how the DC gain and time-constant of a system relate to its pole location, impulse response and step response.
1. In this assignment, we analyze the dynamics of the system. The following differential equations represent the dynamics of the components of the system.

   (i) **Aileron Servo Motor:**
   \[ 0.1\dot{\delta_a}(t) + \delta_a(t) = V_{\text{servo}}(t) \]

   (ii) **Lateral Dynamics of the Aircraft:**
   \[
   \ddot{p}(t) + 1.286\dddot{p}(t) + 0.543\ddot{p}(t) + 0.6114\dot{p}(t) + 0.034p(t) \\
   = 0.23\ddot{\delta_a}(t) + 0.079\dddot{\delta_a}(t) + 0.0789\dot{\delta_a}(t) + 0.0001\delta_a(t)
   \]

   This fourth order system has three modes, two exponential and one oscillatory. The exponential modes correspond to the spiral mode and the roll mode, while the oscillatory mode corresponds to the dutch roll.

   (iii) **Kinematics:** The following equations show how the roll rate \( p(t) \), the bank angle \( \theta(t) \) and the heading \( \psi(t) \) are related:
   \[
   \dot{\phi}(t) = p(t) \\
   \dot{\psi}(t) = \frac{g}{U_0} \phi(t)
   \]
   where \( g = 9.8 m/s^2 \) is the acceleration due to gravity and \( U_0 = 67 m/s \) is the airspeed of the aircraft.

   (iv) **Rate Gyro:** The rate gyro has a sensitivity of \( 0.5V/\text{deg} \), that is, \( V_{\text{gyro}}(t) = 0.5p(t) \)

   Analyze the stability and the nature of the free response of the aileron servo and the lateral dynamics (use MATLAB to solve the fourth order characteristic equation). Write out transfer functions for each of the five equations and draw a block diagram from the input \( V_{\text{servo}}(t) \) to two outputs, \( V_{\text{gyro}}(t) \) and \( \psi(t) \).

2. **(SIMULINK)**
   Creating a subsystem

   1. Open the SIMULINK model that you built in the last assignment by launching SIMULINK from the MATLAB command window and using the *File* menu of the blank model window to open it.

   2. Select all the blocks in the model by using your mouse to draw box around all of them or by using the *select all* option from the *Edit* menu.
3. Use the Create subsystem option of the Edit menu to put all these blocks into one subsystem. Rename the block "Range". You can view the components of the subsystem by double clicking on the block.

We will need this block later in the semester, so make sure that it stays in your SIMULINK model.

Simulating the dynamics of the system

1. Copy two Transfer Fcn blocks from the "Continuous" block library and use them to represent the aileron servo motor and the lateral dynamics of the aircraft. Enter the transfer function coefficients of the numerator and the denominator in the appropriate fields by double clicking on the block.

2. Copy two Integrator blocks from the "Continuous" block library and use them to represent the equations relating \( p(t) \) to \( \phi(t) \) and \( \dot{\psi}(t) \) to \( \psi(t) \). Set the initial value of \( \psi(t) \) to 4 degrees by double clicking on the block.

3. Copy two Gain blocks from the "Math Operations" block library, one to represent the relation between \( \phi(t) \) and \( \dot{\psi}(t) \), and the other for the rate gyro. Set the gain values according to the equations.

4. Connect all these blocks according to the block diagram from the first part of the assignment.

5. Connect To Workspace blocks to the appropriate lines to dump \( \delta_a(t), p(t), \phi(t), \dot{\psi}(t), \psi(t) \) and \( V_{gyro}(t) \) to your MATLAB workspace at the end of your simulation.

6. Copy a Step block from the "Sources" block library and use it as the input to the aileron servo motor.

7. Simulate the system for 200 seconds. Plot and print \( \delta_a(t), p(t), \phi(t), \dot{\psi}(t), \psi(t) \) and \( V_{gyro}(t) \) versus time. Use the MATLAB commands "xlabel" and "ylabel" to label the axes, "title" and "grid". If you are not familiar with these commands, type

\[ \text{>> help command} \]

for online help.

8. Save your model for the following assignment.