1. (16 points) Shown below are the output characteristics of linear and nonlinear multiports. Identify using “E”, “P”, and “F” the points at which maximum effort, maximum power, and maximum flow occur.

2. (21 points) Based on the linear output characteristics shown below, identify by letter the specified points.

- A maximum effort transfer
- B maximum flow transfer
- C maximum power transfer
- B zero effort transfer
- A zero flow transfer
- A infinite load impedance
3. (30 points) Shown below is an electric motor connected to an ideal voltage source. The motor drives a load through a gear train that reduces the motor speed to drive the load. Draw a multiport block diagram of this system showing all of the components and the port variables with the correct causality. Using the mathematics provided, reflect the load to the source. Determine the maximum motor speed and the maximum motor torque. We have already selected the motor and the load. We know that the motor has a maximum efficiency at 75% of its maximum speed (and 25% of its maximum torque).

What gear ratio should you select to allow the motor to run at maximum efficiency?

\[
\text{motor: } T_m = T_m - B_m \omega_m \\
\text{gears: } \omega_L = R_S \omega_m \\
T_m = R_s T_L \\
T_L = B_L \omega_L
\]

**Multiport Equations:**

For the Motor: \( T_m = T_m - B_m \omega_m \) \( \rightarrow \) (1)

For the Gear Train: \( \omega_L = R_S \omega_m \) \( \rightarrow \) (2)

\[ \begin{bmatrix} \omega_L \\ T_m \end{bmatrix} = \begin{bmatrix} R_S & 0 \\ 0 & R_S \end{bmatrix} \begin{bmatrix} \omega_m \\ T_L \end{bmatrix} \]

For the Load: \( T_L = B_L \omega_L \) \( \rightarrow \) (3)

\[ \begin{bmatrix} \omega_L \\ T_m \\ T_L \end{bmatrix} = \begin{bmatrix} T_m - B_m \omega_m \\ R_S \omega_m - B_m \omega_m \\ B_L \omega_L \end{bmatrix} \]

\[ \begin{bmatrix} \omega_m \\ T_m \end{bmatrix} = \begin{bmatrix} \omega_m \\ T_m \end{bmatrix} \]

**O/P Characteristic of Motor (Source):**

\[ T_m \]  \( \rightarrow \) (4)
Reflecting Load to Source

$$T_L = B_L \cdot \omega_L$$

From (2) and (3), we have $\omega_L = R_s \cdot \omega_m$; $T_m = R_s \cdot T_L$

$$\Rightarrow \frac{T_m}{R_s} = B_L \cdot R_s \cdot \omega_m \Rightarrow T_m = \left( B_L \cdot R_s^2 \right) \omega_m \Rightarrow T_m$$

From Graphs (2) and (3), we have:

Max Motor Torque $= T_m^{\omega_m}$

Max Motor Speed:

In $T_m = T_m^{\omega_m} - B_m \cdot \omega_m$, substitute $T_m = 0$

$$\Rightarrow \quad \left( \omega_m \right)_{\max} = \frac{T_m^{\omega_m}}{B_m}$$

We have: $T_m = \left( B_L \cdot R_s^2 \right) \cdot \omega_m$

$$\frac{T_m^{\omega_m}}{4} = \frac{\left( B_L \cdot R_s^2 \right)}{4} \cdot \frac{3 \cdot T_m^{\omega_m}}{B_m}$$

$$\Rightarrow \quad \frac{B_L \cdot R_s^2}{B_m} (3) = 1$$

$$\Rightarrow \quad R_s = \left( \frac{B_m}{3B_L} \right)^{\frac{1}{2}}$$

For the motor to run at max. efficiency, the gear ratio should be:

$$R_s = \left( \frac{B_m}{3B_L} \right)^{\frac{1}{2}}$$
4. (33 points) We want to build a hand pump to inflate automobile tires. The proposed system is shown below. We have measured that an average person can press on the hand pump with a force of $F_0$ (with no velocity) and can move the pump at a velocity of $v_0$ (with no force). It appears that the friction, leakage, and inertia of the pump itself are negligible in comparison to the pressure forces.

The tire has some resistance to flow in the valve of the tire. We have determined that this resistance is linear in terms of the pressure drop and flow characteristics. Further, since the tire volume is large, the internal pressure inside the tire acts as a constant back pressure to the valve during one stroke, $\delta P_t^*$. The pressure-flow characteristics of the valve and tire are shown in the following graph. Notice that the tire pressure is relatively constant for one stroke, but gradually increases with each stroke.

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a. **Draw** the multiport block diagram of the system combining the tire and valve into one block. **Identify** all of the port variables and causalities.

b. **Write** all of the modeling equations in multiport form.

c. **Reflect** the valve characteristic with no pressure in the tire to human's hand output characteristics and show on the graph how the output characteristics vary with the area of the pump piston, $A$. What is the requirement for the area of the pump for an the conditions for maximum power transfer to exist?

d. **Reflect** the hand characteristic to the tire. First consider that the initial pressure in the tire is zero, and show the source characteristic using the optimum area of the pump.

e. Consider that the tire has been pumped to a pressure of $\delta P_t^*$ and you want to pump more pressure into the tire. **Show** what reflected source characteristic would be optimum for maximum power. Now, **derive** the area of the pump that is required.
(work area for problem 4)

Control Systems Components

a) **Source Output Characteristics:**

\[ F = F_0 - Bv \]

For the pump:

\[ \delta P_p = \frac{F}{A} \quad ; \quad Q = Av \]

**Valve characteristic:**

\[ \delta P_p = QR \rightarrow \text{without fire pin}, \delta P_p* + \delta P\rightarrow \text{with } \delta P*_* \]

b) **Modeling Eq. for the hand:**

- \( F = F_0 - Bv \)
- \( \delta P_p = AF \); \( Q = Av \)
- \( \delta P_v = \delta P*_* + QR \)

c) For \( \delta P*_* = 0 \)

Reflecting Load (Valve) to Source (Hand)

\[ \delta P_p = \delta P*_* + QR \quad \Rightarrow \quad \delta P_p = QR \quad \text{C. : } \delta P*_* = 0 \]

\[ \frac{F}{A} = RA^2 \quad \Rightarrow \quad F = (R, A^2) \]

When \( A \rightarrow 0 \)

\[ F = 0 \]

When \( A \rightarrow \infty \)

\[ F = \infty \]
For max power transfer
\[ \left( \frac{P_o}{2} \right) = R_A \left( \frac{P_o}{2R} \right) \Rightarrow A = \left( \frac{B}{R} \right)^{1/2} \]

\[ \Rightarrow \text{For max power transfer:} \quad A = \left( \frac{B}{R} \right)^{1/2} \Rightarrow \text{Opt. Area} \]

d) Reflecting source to load:
\[ F = F_0 - B \frac{Q}{A} \Rightarrow \delta P = \frac{F_0}{A} - \left( \frac{B}{A^2} \right) Q \]

\[ \Rightarrow \delta P = \frac{F_0}{\sqrt{B/R}} - RQ \]

\[ \Rightarrow \delta P = \frac{F_0}{\sqrt{B/R}} \quad \Rightarrow \quad \text{Source Characteristic} \]

e) \[ \delta P = \delta P^* + QR \]
\[ F/A = \delta P^* + AV \cdot R \]
\[ F = \delta P^* \cdot A + A^2 \cdot V \cdot R \]
\[ \frac{F_0}{A} = \delta P^* \cdot A + A^2 \cdot R \cdot \frac{E_0}{2B} \]

\[ \Rightarrow \delta P^* \cdot (A) + A^2 \cdot R = 1 \]

\[ \Rightarrow \left( \frac{R}{B} \right) \cdot A^2 + \left( \delta P^* \right) A - 1 = 0 \]

\[ \Rightarrow A = -\frac{\delta P^*}{2} \pm \sqrt{\left( \frac{\delta P^*}{2} \right)^2 + 4 \left( \frac{R}{B} \right)} \]

The positive value of \( A \) is the optimum area.