1. Find the general solutions to the following linear differential equations:
   (a) \[ \frac{dy}{dt} + y = 10, \]
   (b) \[ \frac{dy}{dt} - 3y = 27, \]
   (c) \[ 4\frac{dy}{dt} + 5y = 100. \]

2. Find the general solutions to the following differential equations:
   (a) \[ t\frac{dy}{dt} + 2y + t = 0, t \neq 0, \]
   (b) \[ \frac{dy}{dt} - \frac{1}{t}y = t, t > 0, \]
   (c) \[ \frac{dy}{dt} - \frac{1}{t^2 - 1}y = t, t > 1, \]
   (d) \[ \frac{dy}{dt} - \frac{2}{t}y + \frac{2a^2}{t} = 0, t > 0. \]
3. Solve the differential equation

\[ 1 + \left( 2 + \frac{t}{y} \right) \frac{dy}{dt} = 0, \quad t > 0, \quad y > 0. \]

4. Solve the following Bernoulli equations assuming \( t > 0, \ y > 0 \):
   
   (a) \[ t \frac{dy}{dt} + 2y = ty^2, \]
   
   (b) \[ \frac{dy}{dt} = 4y + 2e^t \sqrt{y}, \]
   
   (c) \[ t \frac{dy}{dt} + y = y^2 \ln t. \]

5. An economic growth model by Haavelmo (1954) leads to the differential equation

\[ \frac{dK}{dt} = \gamma_1 b K^{\alpha} + \gamma_2 K, \]

where \( \gamma_1, \gamma_2, b, \) and \( \alpha \) are positive constants, \( \alpha \neq 1, \) and \( K = K(t) \) is the unknown function. The equation is separable, but solve it as a Bernoulli equation.

6. A study of the optimal exhaustion of a natural resource uses the equation

\[ \frac{d^2 y}{dt^2} - \frac{2 - \alpha}{1 - \alpha} \frac{dy}{dt} + \frac{a^2}{1 - \alpha} y = 0, \]

where \( \alpha \neq 0, \alpha \neq 1, \) and \( a \neq 0. \) Prove that \( u_1 = e^{at} \) and \( u_2 = e^{at/(1-\alpha)} \) are both solutions. What is the general solution?

7. Find the general solutions of the following equations:
   
   (a) \[ \frac{d^2 y}{dt^2} - 3y = 0, \]
   
   (b) \[ \frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 8y = 0, \]
(c)\[ \frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 8, \]

(d)\[ \frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{5t}, \]

(e)\[ \frac{d^2y}{dt^2} - y = e^{-t}, \]

(f)\[ 3\frac{d^2y}{dt^2} - 30\frac{dy}{dt} + 75y = 2t + 1. \]