Lab 0. Electrostatic charge and potential

Background: Circuits, Chapters 4-6 in Introductory Electromagnetics

Measurement of electrostatic charge and potential is a challenging task for several reasons. First, it is not possible to carry out the measurement of any physical quantity without changing it, at least a little. Second, ordinary measuring instruments have too short a time constant to be able to register potentials long enough to be observed by human senses. Third, voltages needed to produce significant charge distributions are quite high, on the order of kilovolts (kV) or more.

Purpose: (i) to understand how measurements change the quantities they wish to determine, and how we may process the raw measured data to infer what the value of the desired quantity originally was; (ii) to understand the behavior of electrostatic charges and potentials on conducting bodies; and (iii) to understand how electrostatic induction and shielding work.

Pre-lab problems:

PL0.1. The positive electrode of a capacitor holds a total amount of charge $Q = CV$, where $C$ is the capacitance and $V$ the voltage across the capacitor. Suppose we wish to measure this voltage with a DC voltmeter, whose internal resistance is $R_m$. If the voltmeter is connected to a capacitor charged to an initial voltage $V_0$ at the time $t = 0$, what will be the voltmeter reading as a function of time for $t > 0$? If the capacitor value is $C = 100$ pF, will a typical multimeter with $R_m = 10$ MΩ be suitable for measuring the voltage on this capacitor? How large must $R_m$ be in order to make this voltage measurement practical?

PL0.2. A circuit with two capacitors is shown in Fig. L0.1. Suppose that, with the switch open, capacitor $C_1$ has a voltage $V_1 = V_0$, but the voltage across the second capacitor $C_2$ is $V_2 = 0$. After the switch is closed, some charge from $C_1$ is transferred to $C_2$. When steady state is reached, what are the charges $Q_1$ and $Q_2$ on each of the capacitors, and what is the final value of the voltage $V = V_{1f} = V_{2f}$ across both capacitors? You may need to assume that the switch has a small but nonzero resistance in order to solve the problem unambiguously.

PL0.3. Repeat problem PL0.2, but replace the capacitor $C_1$ with a conducting sphere of radius $a$. If the sphere carries a total charge $Q_1$, what is the electrostatic potential (voltage) at the surface of the sphere? What value of capacitance can we attribute to the sphere?

PL0.4. A long line charge with charge density $Q' \text{ C/m}$ is located along the $z$-axis, surrounded concentrically by two long uncharged thin conducting shells of radii $a$ and $b$ ($a < b$) as shown in Fig. L0.2. Using Gauss’ law, obtain an expression for the electrostatic field of this system, and from that obtain an expression for the potential difference between the inner shell and the outer shell (which is taken as the potential reference).
Lab:

Equipment and parts:
- a conducting sphere on an insulated mount;
- a high-resistance voltage source capable of producing up to 3 kV DC;
- a very high-resistance electrometer;
- wands for transferring charge;
- miscellaneous insulated connecting wires and cables;
- a 100 pF capacitor in an insulated mount;
- a Faraday cage.

In your lab report, answer questions L0.1 to L0.11, adding any graphs or sketches that you feel make it clearer. Make sure to label all axes and include scales and units.

Part 1: Calibrating the electrometer

An electrometer is a DC voltmeter with an extremely high input resistance (about $10^{14}$ Ω). When the electrometer is connected to a charged circuit, some of the charge in the circuit is removed to the internal capacitance $C_{em}$ of the electrometer, resulting in a voltage $V_{em}$ being displayed on its meter. Because the internal resistance of the electrometer is so large, the meter reading will remain virtually constant over a long period of time.

L0.1. Charge the 100 pF capacitor to 30 V using the electrostatic voltage source. Remove it from the source, being careful not to touch the capacitor electrodes to yourself or any other object that will allow charge to leak away. Connect a BNC-to-banana-plug adaptor to the electrometer input connector. Press the zeroing button on the electrometer to discharge any residual electrostatic charge that may be present. Connect the capacitor to the banana plug terminals on the electrometer (again being careful not to allow charge to go anywhere except into the meter), and note the voltage reading.

L0.2. From this voltage reading and the knowledge of the initial voltage on the capacitor and its capacitance, what must the internal capacitance $C_{em}$ of the electrometer be? According to the manufacturer, the internal capacitance of the electrometer is 27 pF. Can you give one or more reasons why your result differs from this value?
Calculations of this type are called *calibrations*, and allow us to infer the original values of quantities that are altered by the measurement process. Some instruments can be designed to have an extremely small effect on the quantity being measured, and do not require calibration, but here we see an example where calibration is an essential part of the measurement process.

**Part 2: Measuring charge**

Remove the BNC-to-banana plug adaptor from the electrometer, and connect the BNC-to-alligator-clip test lead assembly. Connect the ground lead to the ground terminal of the electrostatic voltage source.

**L0.3.** Connect the positive terminal of the 30 V electrostatic voltage source to the conducting sphere, and then disconnect it, being careful not to allow charge to be removed from the sphere as you do so. Take care also not to scratch the surface of the sphere (it is plastic with a thin metal plating). Based on this voltage and the fact that the radius of the sphere is \( r = 6.5 \text{ cm} \), from Gauss’ law what should the total charge \( Q_i \) on the sphere be? What is the surface charge density \( \sigma_i \) on the sphere?

**L0.4.** Zero the electrometer, then touch its positive lead to the sphere and remove it. What is the voltage reading? How much charge \( Q_{em} \) was transferred to the electrometer in taking this measurement, based on your knowledge of the electrometer capacitance \( C_{em} \) from step **L0.2**?

**L0.5.** Calculate the charge \( Q_f = Q_i - Q_{em} \) remaining on the sphere after the measurement in step **L0.4**. Does the measured electrostatic potential agree with the value obtained for the sphere with this charge using Gauss’ law?

**Part 3: Electrostatic shielding and induction**

Many materials encountered in our daily lives are *triboelectric*; i.e., they can transfer charges to each other by means of friction. Examples of these are certain fabrics, rubber and other insulators of various kinds. As we walk across a carpeted floor, for instance, our bodies can accumulate excess charge. If we were then to touch a piece of electronic equipment without removing that charge first, a brief but intense current could result that is potentially damaging to the semiconductor devices in the circuit. That is why you use an antistatic grounding strap when working with such equipment. In the next step, you will demonstrate the presence of electrostatic charge in the environment.

**L0.6.** Connect the positive terminal of the electrometer directly to the terminal on the sphere using the BNC-to-spade-lug cable. The ground terminal of the electrometer can be connected to a metal portion of the lab bench, a power-line ground, or left disconnected. Zero the electrometer, and set it to its most sensitive range. Observe what happens as you move your body in the vicinity of the sphere, or move the sphere relative to the bench. Explain qualitatively what is happening.

A *Faraday cage* is an arrangement of one or more mesh conductors in such a way as to provide shielding or to exhibit electrostatic induction. The holes in the mesh are small enough that the meshes can be considered to behave as if they were continuous conductors. Our Faraday cage has two nested conducting meshes: the inner one has a radius of 5 cm, while the outer one has a radius of 7.5 cm as shown in Fig. **L0.3**. We will use this Faraday cage to demonstrate electrostatic induction.

**L0.7.** Disconnect the electrometer from the conducting sphere. Connect the positive terminal of the electrometer to the inner mesh of the Faraday cage, and the ground terminal to the outer mesh using the BNC-to-alligator-clip cable. Discharge any remaining charge on the electrometer by zeroing it, and from the Faraday cage by grounding it, repeatedly if necessary. Connect the sphere to a 3 kV source (this source has a high internal resistance, and so should not pose a danger to you). **NOTE:** Do NOT connect this high voltage source directly to the electrometer—it’s maximum allowed voltage is ±100 V. According to Gauss’ law, what should be the surface charge density \( \sigma \) on the surface of the sphere?
\[ h = 15 \text{ cm} \]
\[ d_1 = 15 \text{ cm} \]
\[ d_2 = 10 \text{ cm} \]

**Fig. L0.3:** Double-layer Faraday cage.

**L0.8.** Take the wand with the silver-colored pad and touch it tangentially (flat) against the surface of the sphere, then remove it. Assume that the pad removes all the charge from the sphere that is directly under its area (the diameter of the pad is 3 cm). How much charge is there on the pad? Carefully insert the silver pad inside of the inner mesh of the Faraday cage, without touching it to either of the meshes or anything else. What is the voltage reading on the electrometer?

**L0.9.** Using the result of prelab homework problem PL0.4, what total amount of charge on a 15 cm long line charge along the axis of the two meshes would be necessary to produce the voltage measured in step L0.8? Compare this to the amount of charge computed in step L0.8. How do these values compare? Suggest possible reasons for the difference.

**L0.10.** Discharge any remaining charge from the electrometer and Faraday cage as in step L0.7. Take the wands with the blue and white pads at the ends, and rub the faces of these pads together for several seconds. The materials in these pads are triboelectric. Carefully insert the blue pad inside the inner mesh of the Faraday cage, without touching it to either of the meshes or anything else. What is the voltage reading on the electrometer? Calculate the amount of charge on the blue pad by the method of step L0.9. Was this charge positive or negative?

**L0.11.** Repeat L0.10 using the wand with the white pad.

**Conclusions:**

1. The act of measuring electrostatic charge and potential necessarily changes these quantities. The measurement technique must account for this by suitable post-processing of the measured data. This process is called calibration.

2. Charge can be transferred from one conducting object to another. Charge can also be moved between two initially uncharged insulating objects.

3. Most objects carry a certain amount of electrostatic charge under normal circumstances.

4. The potential between uncharged conductors can be used to detect the charge placed inside them.