Class 22 – Fluid Mechanics

Thursday, April 7th

External flow characteristics

- Fix coordinate system so that fluid flows past a stationary body with velocity $U$
- Classify bodies as:
  - 2D, axisymmetric, 3D
  - Streamlined or blunt
- Function of Re, Ma, Fr

Streamlined vs. blunt

The resultant force in the direction of the upstream velocity is:

a) Drag
b) Lift
c) Depends on the shape of the object
Which has the highest Re?

a)  
b)  
c)  

Boundary layers

\[ Re_{BL} = 5 \times 10^5 \]  
\[ \delta \text{ is when } Re < 990 \]  
transition
Boundary Layer Thicknesses

- Disturbance Thickness, $\delta$
- Displacement Thickness, $\delta^*$
  \[ \delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy \]
- Momentum Thickness, $\theta$
  \[ \theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy \]

Laminar Flat-Plate Boundary Layer: Exact Solution

Governing Equations

- $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

- Compressible velocity normal
- $\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = y \frac{\partial^2 u}{\partial y^2}$

- $\frac{\partial}{\partial x} \ll \frac{\partial}{\partial y}$

Laminar Flat-Plate Boundary Layer: Exact Solution

Boundary Conditions

- $y = 0, \quad u = 0, \quad v = 0$
- $y = \infty, \quad u = U, \quad \frac{\partial u}{\partial y} = 0$
Laminar Flat-Plate Boundary Layer: Exact Solution

- Equations are Coupled, Nonlinear, Partial Differential Equations
- Blasius Solution:
  - Transform to single, higher-order, nonlinear, ordinary differential equation
  \[ \frac{2}{\eta} \frac{d^2 f}{d\eta^2} + f \frac{d^2 f}{d\eta^2} = 0 \]
  \[ \eta = \infty, \quad \frac{df}{d\eta} = 1 \]

Water flows past a flat plate with an upstream velocity of \( U = 0.02 \text{ m/s} \). Determine the water velocity a distance of 10 mm from the plate at distances of \( x = 1.5 \text{ m} \) and \( x = 15 \text{ m} \) from the leading edge.

Which of the following is true?

a) \( u_1 > u_2 \)
b) \( u_1 < u_2 \)
c) \( u_1 = u_2 \) \( \eta \cdot \left( \frac{u}{U} \right)^2 \eta \)

\[ \eta = \frac{5 \delta}{\sqrt{f'U}} = \frac{3 \delta}{\sqrt{Re}} \]
\[ \delta^* = \frac{1.721}{\sqrt{Re}} \]
\[ \frac{\theta}{x} = c_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} = \frac{0.864}{\sqrt{Re}} \]

\[ C_{f\theta} = \int_0^l \tau_w dx = 1.328 \]

<table>
<thead>
<tr>
<th>TABLE 9.1</th>
<th>Laminar Flow along a Flat Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta = \frac{y(U/\nu)^{1/2}}{\nu} )</td>
<td>( f'(\eta) = U/\nu )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1328</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2647</td>
</tr>
<tr>
<td>1.2</td>
<td>0.3938</td>
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<tr>
<td>1.6</td>
<td>0.5168</td>
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</tbody>
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Water flows past a flat plate with an upstream velocity of $U = 0.02$ m/s. Determine the water velocity a distance of 10 mm from the plate at distances of $x = 1.5$ m and $x = 15$ m from the leading edge.

\[
\eta = x(U/\rho g)^{1/2}, \quad f(\eta) = u/U
\]

\[
\begin{array}{c|c}
0 & 0 \\
0.4 & 0.1328 \\
0.8 & 0.2647 \\
1.2 & 0.3934 \\
1.6 & 0.5168 \\
2.0 & 0.6298 \\
2.4 & 0.7290 \\
2.8 & 0.8143 \\
3.2 & 0.8761 \\
\end{array}
\]

\[
u_1 = 0.0074 \gamma_1 \gamma_2 \quad u_2 = 0.0027 \gamma_2 \gamma_2
\]

A smooth plate of length $l = 6$ m and width $b = 4$ m is placed in water with an upstream velocity of $U = 0.5$ m/s. Determine the boundary layer thickness and the wall shear stress at the center and the trailing edge of the plate. Assume laminar BL.

\[
\tau_w \approx 2 \frac{f(\eta)}{\sqrt{\eta}} \frac{U}{\sqrt{\nu}} = 0.124 \frac{U}{\nu}
\]

\[
\frac{\tau_w}{\frac{U}{\nu}} \approx 0.532 \frac{U}{\nu}
\]

What if it’s a cylinder instead of a flat plate in viscous flow?

a) The free-stream velocity is constant.

b) The pressure field is not uniform.

c) Both a and b.
What if there is a pressure gradient?

\[
\frac{dp}{dx} = -\rho U_\mu \frac{dU_\mu}{dx}
\]

\[
\tau_\nu = \rho \frac{dU_\mu \theta}{dx} + \rho \delta \epsilon U_\mu \frac{dU_\mu}{dx}
\]